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A NEW METHOD OF RC THREE - TERMINAL NETWORK TRANSFER FUNCTION SYNTHESIS

A NEW METHOD OF RC THREE - TERMINAL NETWORK
TRANSFER FUNCTION SYNTHESIS

A thesis submitted in partial fulfillment
of the requirements for the degree of
Doctor of Philosophy

By

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INTRODUCTION

Control system compensation and optimization, signal/noise separation, and system simulation are some areas in which the following problem must frequently be solved: Given a real rational algebraic function, construct a physical system which has its input-output relationship described in the complex plane by the given function. In other words, construct a physical system which has a given transfer function. If the system to be constructed is to be an electrical network, then this problem falls into the area of network synthesis - specifically, the area of network realization techniques.

Of particular interest in the above mentioned applications is the voltage transfer function T(S) - the ratio of steady state output voltage to input voltage in the domain of the complex frequency variable S. In many cases it is desirable to realize a given T using only two kinds of passive elements and no active devices. For example, in low frequency servomechanisms resistor-capacitor (RC) compensating networks are usually desired. Another frequently required feature is the existence of a common reference node or ground between the input and output terminals of the network.

It is this very special type of network, the RC three-terminal network, with which this thesis is concerned. The primary purpose of the thesis is to provide a new procedure for constructing an RC network having certain terminal characteristics (e.g. T) specified beforehand. The realization procedure to be presented gives the network designer the tools to develop a variety of networks for a given set of network specifications. During the development the designer exercises much control over such features of the resulting network as the network structure, the gain, and the number of elements.

The development of a network is carried out using the parameters y_{22}, y_{21}, and T. These parameters are defined in the usual manner in Section 1.1. Section 1.2 gives the conditions which a given set of these parameters must satisfy if they are to characterize an actual RC three-terminal network. Any RC network realization technique is easily extended to the RL or LC class of networks. Section 1.3 presents one method of reducing RL and LC realization problems to RC problems.

The tools of the realization procedure are four types of network removals. The designer must choose one of these four during each step of the development. In Chapter 2 each type of removal is discussed in detail with emphasis placed upon what can be accomplished by a particular removal when it is used during the development of a network. Examples are given to illustrate effective use of the various types of removals and to illustrate the ease with which development of a network can be carried out.

Examination of the bibliography indicates that there are many methods which might be employed to develop an RC network given certain terminal characteristics. Several examples, solved previously by other methods, are presented in Chapter 3 to provide a comparison of networks obtained by the method of Chapter 2 to those obtained by the other methods.

The elements in all circuit schematics of this thesis are given in units of conductance (not resistance) and capacitance. This is mentioned here in the introduction to avoid noting it on each schematic presented.

CHAPTER 1

THE NETWORK PARAMETERS

1.1 Definitions.

This thesis is concerned primarily with three-terminal networks (3 T.N.). A 3 T.N. is a special case of the two-port or two-terminal-pair network (i.e. one terminal of each pair is a common point or ground point of the network). Figure 1-1 illustrates a general 3 T.N. together with the conventional assignment of voltages and currents which can be measured at the terminals.

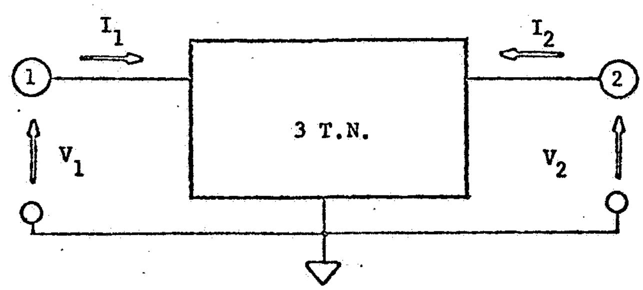

Figure 1-1. A three-terminal network.

Description of the terminal characteristics of a linear 3 T.N. can be accomplished through use of the short-circuit admittance parameters which are defined by the following relationships:

$$I_1 = y_{11} V_1 + y_{12} V_2 , \qquad (1\text{-}1)$$

$$\text{and:} \quad I_2 = y_{21} V_1 + y_{22} V_2 \,. \tag{1-2}$$

The thesis is especially concerned with the driving point admittance:

$$y_{22} = \frac{I_2}{V_2}\bigg|_{V_1=0} , \tag{1-3}$$

the transfer admittance[1]:

$$y_{21} = \frac{I_2}{V_1}\bigg|_{V_2=0} , \tag{1-4}$$

and the forward voltage transfer function:

$$T = \frac{V_2}{V_1}\bigg|_{I_2=0} = \frac{-y_{21}}{y_{22}} \,. \tag{1-5}$$

Each of these expressions follows immediately from Eqn. 1-2.

Assume for the moment that it is possible to construct a network having these and only these parameters (i.e. y_{22}, y_{21}, and T) specified. Then just what type of transfer characteristics (i.e. input-output relationships) can be realized? Two obvious answers are: (1) an input voltage-open circuit output voltage relationship (i.e. T); or (2) an input voltage-short circuit output current relationship (i.e. y_{21}). Not so apparent an answer is: (3) an input current-short circuit output current relationship. This is possible since from Eqns. 1-1 and 1-2:

$$\frac{I_1}{I_2}\bigg|_{V_1=0} = \frac{y_{21}}{y_{22}} \,. \tag{1-6}$$

Since I_2 is the input, the resulting network would have to be turned

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[1]In all networks to be discussed the reciprocity property holds; that is $y_{21} = y_{12}$. Only y_{21} will appear hereafter.

around in order to conform with the convention of connecting the input source to the left hand side terminals.

1.2 Parameter Restrictions.

The next important question to be answered is "What restrictions must be placed upon these parameters in order to guarantee that they represent an RC 3 T.N.?".

The RC Driving Point Admittance. First the parameter y_{22} will be considered. The admittance y_{22} is a driving point admittance (i.e. the current to voltage ratio taken at one pair of nodes), and in this thesis it is an RC driving point admittance. It is well known that an RC driving point admittance y can always be written as:

$$y(S) = C \frac{(S+a_1)(S+a_2)\ \dots\ (S+a_m)}{(S+b_1)(S+b_2)\dots(S+b_n)} \qquad (1\text{-}7)$$

where:
(1) $0 \leq a_i < a_{i+1}$; $i = 1,2,\dots,m-1.$
(2) $0 < b_i < b_{i+1}$; $i = 1,2,\dots,n-1.$
(3) $a_i < b_i < a_{i+1}$; $i = 1,2,\dots,n.$
(4) $C > 0.$
(5) Either $m = n$ or $m = n+1.$

These conditions can be established from Foster's reactance theorem [15] (LC driving point criteria) by a simple LC $\rightarrow$ RC transformation. A thorough discussion of LC and RC driving point admittances can be found in most introductory network synthesis textbooks. A relatively simple, yet somewhat less known, derivation of the necessity of Foster's conditions has been given by Tsang [33].

The sufficiency of these conditions can be established by expanding the function y/S into partial fractions and then multiplying through by S; this gives:

$$y(S) = C_\infty S + G_0 + \sum_{i=1}^{n} \frac{G_i S}{S+b_i}. \qquad (1\text{-}8)$$

Calculation of C_∞, G_0, and G_i from y given in Eqn. 1-7 will always yield non-negative real numbers. Eqn. 1-8 can be recognized as the parallel combination of a capacitor C_∞, a conductance G_0, and n admittances each consisting of a conductance G_i in series with a capacitance G_i/b_i.

<u>The RC Voltage Transfer Function</u>. Let us now consider the transfer function T(S). From analysis experience it is known that T will be a ratio of real polynomials in S:

$$T(S) = K \frac{S^m + c_{m-1}S^{m-1} + \dots + c_0}{S^n + d_{n-1}S^{n-1} + \dots + d_0}. \qquad (1\text{-}9)$$

The necessary and sufficient restrictions to be placed upon Eqn. 1-9 in order for it to represent an RC 3 T.N. voltage transfer function were established by Fialkow and Gerst [9]. The necessity of their conditions will now be derived in a different manner.

Assume that an RC 3 T.N. is given and through methods of circuit analysis y_{22}, y_{21}, T, and z_{11} are calculated. The impedance z_{11} is the network input impedance with $I_2 = 0$; consequently, along with y_{22}, $1/z_{11}$ must be an RC driving point admittance. Network analysis also provides the following equality:

$$z_{11} = \frac{y_{22}}{\Delta y}\,, \quad \text{where: } \Delta y = y_{11}y_{22} - y_{21}^{2}\,. \tag{1-10}$$

Equation 1-10 can be inverted and written as:

$$\frac{1}{z_{11}} = y_{11} - \frac{y_{21}^{2}}{y_{22}}\,, \tag{1-11}$$

or:

$$y_{11} - \frac{1}{z_{11}} = \frac{y_{21}^{2}}{y_{22}}\,. \tag{1-12}$$

Dividing Eqn. 1-12 by y_{22} and using Eqn. 1-5 gives:

$$\left(\frac{1}{y_{22}}\right)\left(y_{11} - \frac{1}{z_{11}}\right) = T^{2}\,. \tag{1-13}$$

Let p_j be any pole of T; hence, it is at least a double pole of T^2. Equation 1-13 can be multiplied by $(S-p_j)^2$ and evaluation of the result at $S=p_j$ can be attempted:

$$\left(\frac{S-p_j}{y_{22}}\right)\left((S-p_j)y_{11} - \frac{S-p_j}{z_{11}}\right)\Bigg|_{S=p_j} = (S-p_j)^2T^2\Big|_{S=p_j}\,. \tag{1-14}$$

Each of the expressions:

$$\frac{S-p_j}{y_{22}}\Bigg|_{S=p_j}\,, \quad (S-p_j)y_{11}\Big|_{S=p_j}\,, \text{ and } \frac{S-p_j}{z_{11}}\Bigg|_{S=p_j}$$

must be finite since y_{22}, y_{11}, and z_{11} have only simple poles and zeros as required by Eqn. 1-7. Therefore, the left and consequently the right hand side of Eqn. 1-14 is finite. Hence, p_j is a simple pole of T, which implies that each pole of T is a simple pole.

Furthermore, since p_j is a pole of T, $(S-p_j)T\big|_{S=p_j}$ is not zero. The left hand side of Eqn. 1-14 contains the factor $\left((S-p_j)y_{11} - \frac{S-p_j}{z_{11}}\right)\Big|_{S=p_j}$, which is finite. Consequently, the factor

$\left.\frac{S-p_j}{y_{22}}\right|_{S=p_j}$ cannot equal zero. If it were zero, the equality in Eqn. 1-14 would not hold since it has been established that the right hand side is not zero. Thus:

$$\left.\frac{S-p_j}{y_{22}}\right|_{S=p_j} \neq 0 , \quad \text{or} \quad y_{22}(p_j) = 0. \qquad (1\text{-}15)$$

Therefore, each pole of T must be real and non-positive since it is a zero of y_{22}.

Summarizing, it can be concluded that poles of T are real, non-positive, and distinct. This statement can be strengthened by noting that T(0) must be finite since it corresponds to the network steady state direct current (d.c.) gain. Hence, poles of T are negative and distinct.

Let us now examine the network with $S = \sigma$ where $\sigma \geq 0$, as suggested by Lewis [21]. Since each element now appears to be a non-negative conductance, the network is equivalent to an all resistor network which has for its voltage gain (given any σ above) the number $T(\sigma)$. It is impossible for a network composed only of positive resistors to have a voltage gain greater than unity. Furthermore, it can equal unity (except in the trivial case when $T \equiv 1$) only when $\sigma = 0$ or $\sigma = \infty$ (i.e. when capacitors become open or short circuits). Similarly, the gain can equal zero (except in the trivial case when $T \equiv 0$) only when $\sigma = 0$ or $\sigma = \infty$. These conclusions can be stated as:

$$0 \leq T(\sigma) \leq 1 \quad \text{for} \quad 0 \leq \sigma \leq \infty , \qquad (1\text{-}16)$$

$$\text{and:} \quad 0 < T(\sigma) < 1 \quad \text{for} \quad 0 < \sigma < \infty . \qquad (1\text{-}17)$$

Equation 1-16 requires that $m \leq n$, and that $K < 1$ whenever $m = n$. Evaluating T at $\sigma = 0$ gives:

$$(Kc_0/d_0) \leq 1 \ , \quad \text{or when } c_0 \neq 0\text{:} \quad K \leq d_0/c_0. \quad (1\text{-}18)$$

Equation 1-17 prohibits T from having zeros on the positive real axis. Furthermore, if k_d is the minimum value of $K/T(\sigma)$ on $0 < \sigma < \infty$ (whenever the minimum exists), then Eqn. 1-17 establishes the requirement $K < k_d$. It is also apparent that $K > 0$.

If the numerator of Eqn. 1-9 is rewritten as an n^{th} degree polynomial (the first n-m coefficients are actually zero), and if the above conditions which T must satisfy are summarized, then Eqn. 1-19 is the result.

$$T(S) = K \frac{c_n S^n + \ldots + c_0}{d_n S^n + \ldots + d_0}, \text{ where } d_n \neq 0. \quad (1\text{-}19)$$

(1) Poles of T are negative and distinct.

(2) No zero of T lies on the negative real axis.

(3) $0 < K \leq \min(d_n/c_n \ , \ d_0/c_0)$.

(4) If $k_d = \min\limits_{0<\sigma<\infty}(K/T(\sigma))$ exists, then $K < k_d$.[1]

Note that the leading coefficients (c_m and d_n) of the numerator and denominator polynomials of Eqn. 1-19 are not necessarily unity. This introduces the possibility of normalizing the upper bound on K as determined by (3) and (4) above. For example, a given T could be rewritten with each of the numerator coefficients

---

[1] If $T = KN(S)/D(S)$, k_d may alternately be defined as the least value of k for which $D(\sigma) - kN(\sigma)$ has a zero on $\sigma > 0$. If σ_0 is this zero, then $k_d = D(\sigma_0)/N(\sigma_0)$ which is the minimum value of $K/T(\sigma)$.

having been multiplied by the constant C. The upper bound on K for the new T would be 1/C times the original upper bound on K. If C is chosen equal to the original upper bound, then the upper bound on K for the new T is unity. Later this operation will be referred to as scaling the upper bound on K.

The sufficiency of the conditions in Eqn. 1-19 is best established by giving a general realization procedure for any T which satisfies Eqn. 1-19. The author is aware of only one such procedure, that of Fialkow and Gerst [9]. This procedure is briefly discussed in Section 3.1. One noteworthy conclusion that can be drawn from Eqn. 1-19 (2) is that distinct common (surplus) factors of the type (S+a), $a > 0$, can be introduced into the numerator and denominator of T until the numerator has all non-negative coefficients. A proof of this is given in [9] since it is necessary to do this in order to perform their synthesis. Hereafter, it will be assumed that this modification has been made to T. That is:

$$c_i \geq 0 \quad \text{for} \quad i = 0,1,\ldots,n. \tag{1-20}$$

From analysis it is known that the calculation of T from a given network involves only multiplication and addition of admittances (e.g. T could be calculated by eliminating all internal network nodes via repeated application of the general star-mesh transformation [21] [32]). Consequently, if cancellation of common factors is prohibited during the calculation of T, then the resulting T must have non-negative coefficients. By interchanging input and ground terminals, it is also true that 1-T is a 3 T.N. voltage transfer function. Thus, it must also have non-negative coefficients.

This gives: $$d_i - Kc_i \geq 0 \quad \text{for } i = 0,1,\ldots,n. \qquad (1\text{-}21)$$

As a necessary condition, Eqn. 1-19 (3) and (4) can be replaced with the more usable criterion:

$$0 < K \leq \min_{\forall i}(d_i/c_i). \qquad (1\text{-}22)$$

Equation 1-22 cannot be used to calculate the upper bound on the gain associated with a given pole-zero configuration for T. It can be used, however, to predict the maximum possible K under the condition of any given surplus factors. The following example will illustrate this.

Example 1-1. Consider the transfer function:

$$T = K\,\frac{(S+1)(S+6)}{(S+2)(S+3)} = K\,\frac{S^2+7S+6}{S^2+5S+6}.$$

Equation 1-22 gives a number which K cannot exceed:

$$K \leq \min\left(\frac{6}{6},\frac{5}{7},\frac{1}{1}\right) = \frac{5}{7}.$$

If a surplus factor of (S+1) is introduced into T (thereby increasing the complexity of the network), a larger K is then indicated:

$$T = K\,\frac{S^3+8S^2+13S+6}{S^3+6S^2+11S+6},$$

which gives: $$K \leq \min\left(\frac{6}{6},\frac{11}{13},\frac{6}{8},\frac{1}{1}\right) = \frac{3}{4}.$$

No matter what surplus factors are introduced, it is certain that K cannot violate the restriction dictated by Eqn. 1-19(4). $K/T(\sigma)$ is a minimum when: $(\sigma^2+5\sigma+6)(2\sigma+7)-(\sigma^2+7\sigma+6)(2\sigma+5) = 0.$ This is true when $\sigma = \sqrt{6}$. Hence:

$$k_d = \frac{K}{T(\sqrt{6})} = \frac{5+2\sqrt{6}}{7+2\sqrt{6}} \cong 0.832 \quad \text{and} \quad K < k_d \cong 0.832.$$

A Summary of Restrictions. It is now possible to summarize the conditions which must be placed on the set of parameters (y_{22}, y_{21}, and T) in order that they represent an RC 3 T.N.

(1) T(S) must satisfy Eqn. 1-19 (as modified by Eqn. 1-20).

(2) Each pole of T(S) must be a zero of y_{22} (Eqn. 1-15).

(3) y_{22} must be an RC driving point admittance (Eqn. 1-7).

(4) $-y_{21}$ must equal T y_{22}, and the coefficients of the numerator polynomial of $-y_{21}$ must be non-negative. Note that each pole of y_{21} is a pole of y_{22}.

It should be noted that each zero of y_{22} need not be a pole of T (i.e. y_{21} could also have the same zero). Zeros common to y_{22} and y_{21} will be called common zeros. Note also that each pole of y_{22} need not be a pole of y_{21}. A pole of y_{22} which is not a pole of y_{21} will be called a private pole.

1.3 Two-Element-Kind Network Transformations.

RC network realization procedures are easily generalized to include other classes of networks containing only two kinds of passive two-terminal elements. That is for each RC network there is a related RL and LC network. Conversely, for each RL or LC network there is a corresponding RC network.

Consider any RC network. It is an interconnection of elements having one of two types of admittance: a conductive admittance which can be specified by a constant such as g; or a capacitive admittance specified, for example, by CS. If each network admittance is scaled by the factor 1/S, then the RC network becomes an RL network. For instance, g becomes g/S which is an inductive admittance; and CS becomes C which is a conductance. The terminal admittances of the RL network equal the terminal admittances of the corresponding RC network multiplied by 1/S. Note, however, that this RC $\rightarrow$ RL transformation does not affect T. Equation 1-5 shows that T for the RL network is identical to T for the RC network. The inverse transformation (i.e. RL $\rightarrow$ RC) is accomplished by multiplying the RL admittances by S.

The above transformation is not the only way to show correspondence between RL and RC networks (e.g. replacing S by 1/S carries an RC into an RL network and vice versa). The purpose of this section is to establish equivalence of the RL (or LC) synthesis problem and the RC problem, thereby justifying the study of only RC realization procedures. An RC-RL equivalence has been established, so it is unnecessary to discuss additional transformations.

One way to establish correspondence between RC and LC networks is to replace S in the RC admittances with S^2 and to divide the result by S. Capacitors remain capacitors, but resistors become inductors. RC $\rightarrow$ LC is thus accomplished by:

$$SC \rightarrow S^2C/S = SC \, , \qquad g \rightarrow g/S = 1/LS, \quad \text{and:}$$

$$T(S) \rightarrow \frac{-y_{21}(S^2)/S}{y_{22}(S^2)/S} = T(S^2) \, .$$

The inverse transformation (i.e. LC→RC) is accomplished by multiplying the LC admittances by S and then replacing S^2 with S.

The following conclusion can be drawn from the above discussion: Given a set of RL or LC network parameters, a network can be developed using an RC network realization procedure. The following steps should be taken:

(1) Transform the parameters into RC network parameters.

(2) Realize the RC network.

(3) Change the RC elements to appropriate RL or LC elements as governed by the inverse of the original transformation.

This procedure is summarized in Figure 1-2 for the two transformations discussed above.

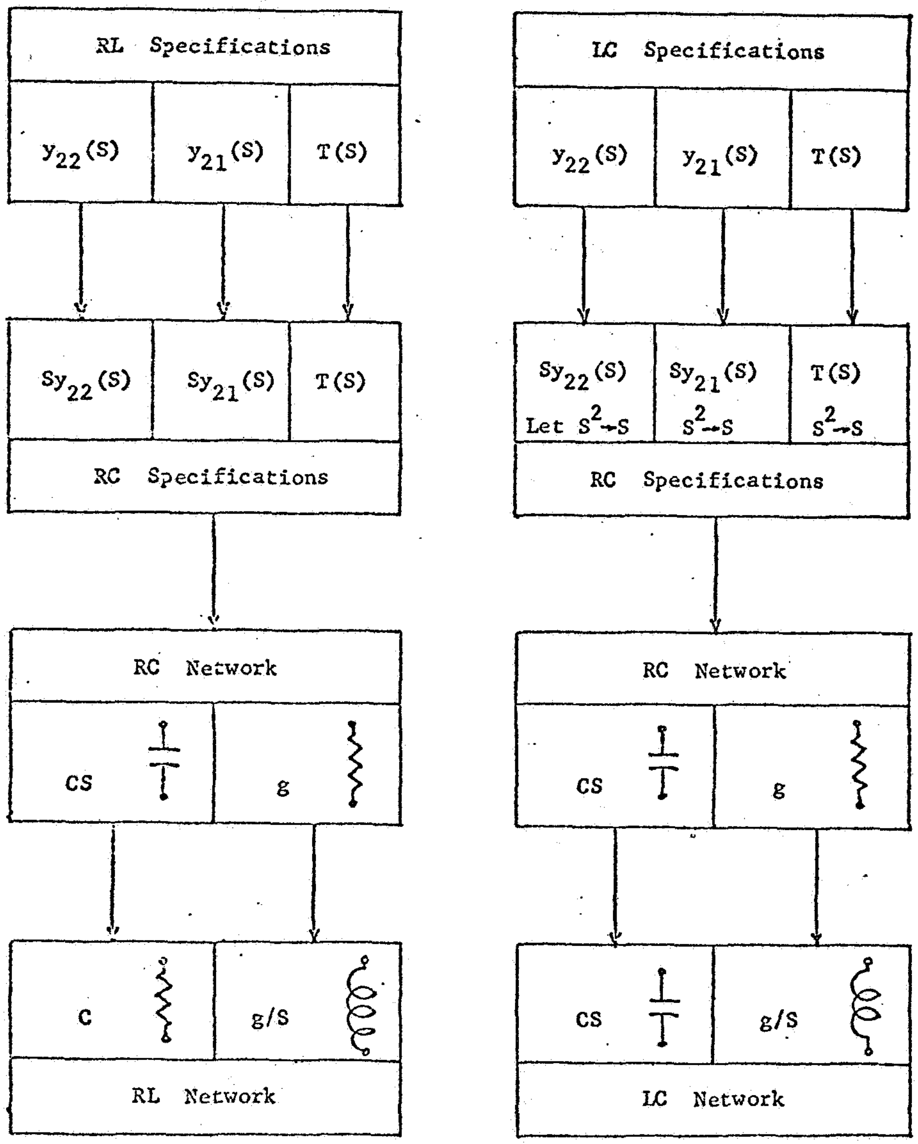


Figure 1-2. A method of RL and LC synthesis using RC realization procedures. Elements are specified in admittance format.

CHAPTER 2

THE REALIZATION PROCEDURE

This chapter is concerned with constructing a network which is to have a given set of terminal characteristics (y_{22}, y_{21}, and T). It is assumed, of course, that the given set of parameters represent an RC 3 T.N. (i.e. satisfy the restrictions of Section 1.2). Let N be the network to be realized. The basic idea of the method to be presented is to realize part of N such that when this partial realization is properly connected to a network N', the total result will be the network N. More specifically, an admittance is removed from N in such a manner that the remaining network to be realized, N', is less complex than N. Subsequent removals result in further simplification of the remaining network until the total realization is finally completed.

Tabulation. As the development of a given set of parameters into a network is carried out, it is quite helpful to have a systematic tabulation of the steps which are taken. After each removal, the parameters of the remaining network to be realized are calculated. Statement of these parameters actually defines a new problem which, if progress is being made, is an easier realization problem than the preceding one. A good table should thus provide direct observation of the progress being made. An extension of Tsang's table [31], which was originally proposed for the zero-shifting technique, will be used.

Consider the table shown in Figure 2-1. It is divided into two main sections entitled 'Network Parameters' and 'Removals'. The parameters describing the desired network are entered on the first line in their respective columns. To the right of these, in the removal section but still on line one, pertinent data about the first removal is entered. Specifically, the admittance (y) of the removal and the manner in which it is to be removed (the 'type' of removal) are entered. As will be seen later, this information will permit direct calculation of the parameters of the remaining network to be realized. These parameters are then entered on line two, which actually states a new realization problem.

The 'N' column is used to designate which network is being developed when there is more than one network development in a single table. A need for this column does not arise until Section 1.3. A complete explanation of its use is given in Section 1.3.

Network Parameters			Removals		
y_{22}	$-y_{21}$	T	y	type	N

Figure 2-1. A table for tabulating the network development.

Network Gain. The method of this chapter does not always realize the network parameters y_{21} and T exactly. The pole-zero configurations of T and y_{21} are realized as specified, but the overall network functions are realized only to within a constant multiplier, K, which is easily determined at the completion of the development. Hereafter, whenever the parameters are specified, the factor K will appear in both T and y_{21}.

It is normally desirable to have as large a network gain as possible. An upper bound, however, was placed upon K in Section 1.2. If the upper bound is determined by requirement (3) of Eqn. 1-19, then the maximum K can usually be attained. Specifically, a network d.c. gain or infinite frequency gain of unity can be attained by avoiding resistive paths to ground in the former case, and avoiding capacitive paths to ground in the latter case.

It is quite easy to realize relatively low gains exactly. Assume for the moment that a network has been developed, and for this realization K is determined to be the number A. Then by merely modifying the final step in the development, any K such that $0 < K \leq A$ can be attained exactly. However, this gain reduction will usually require one (sometimes more) additional element. A discussion of the gain reduction procedure will be given in Section 2.1 (see the subsection entitled The Network Gain Constant).

2.1 Shunt and Series Removals.

Shunt Removal. The simplest type of removal from the network N specified by y_{22}, y_{21}, and T is the shunt (Sh) admittance y, removed as shown in Figure 2-2. The parameters of the remaining net-

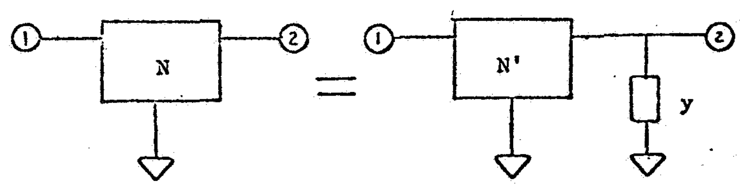


Figure 2-2. The shunt removal.

work N' are easily calculated from:

$$y_{22}' = y_{22} - y \,, \tag{2-1}$$

$$-y_{21}' = -y_{21} \,, \tag{2-2}$$

and

$$T' = -y_{21}/y_{22} \,. \tag{2-3}$$

What type of admittance should be removed in order to make the primed parameters a simpler set than the original set of parameters? Remember that N' must be an RC 3 T.N., so the primed parameters must satisfy the conditions expressed in Section 1.2.

The admittances y and y_{22} are both RC driving point admittances and can therefore be expanded into the form of Eqn. 1-8:

$$y_{22} = C_\infty S + G_0 + \sum_i \frac{G_i S}{S+b_i} \,, \tag{2-4}$$

$$\text{and} \qquad y = c_\infty S + g_0 + \sum_i \frac{g_i S}{S+p_i} . \qquad (2\text{-}5)$$

Subtracting Eqn. 2-5 from 2-4 gives y_{22}' per Eqn. 2-1. The partial fraction expansion of y_{22}'/S must yield all non-negative coefficients. This requires that each pole of y must be a pole of y_{22}. Then:

$$y_{22}' = (C_\infty - c_\infty)S + (G_0 - g_0) + \sum_i \frac{(G_i - g_i)S}{S+b_i}, \qquad (2\text{-}6)$$

where: $c_\infty \leq C_\infty$, $g_0 \leq G_0$, and $g_i \leq G_i$

are further restrictions to be placed upon y.

First it is noted that a pole of y_{22}' need not be a pole of y_{22}; the pole at $S=-b_i$ can be eliminated by making $g_i = G_i$. By recalling that each pole of y_{21}' must be a pole of y_{22}' and by noting Eqn. 2-2, it can be concluded that complete removal of a pole of y_{22} is permissible only when the pole is a private pole. Furthermore, it is usually desirable to eliminate private poles, since y_{22}' and T' are then less complex than y_{22} and T.

What else can be accomplished by a shunt removal? Note in Eqn. 2-6 that the poles of y_{22}' are all poles of y_{22}, but that the zeros of y_{22}' will in general be quite different from the zeros of y_{22}. Hence, the shunt removal can be used as a zero-shifting step.

It is sometimes possible to use the zero-shifting property to simplify the realization of N. For example, assume that the zeros of y_{22} are shifted in such a manner that y_{22}' and y_{21}' have a common zero (this is possible only if y_{21} has a real non-positive zero). Now if y_{21} has no right half plane zeros, then cancellation of the common zero can occur during the calculation of T' from Eqn. 2-3. The resulting N' is less complex than N since the order or degree (i.e. the degree of the denominator) of T' is lower than the order of T. However, if y_{21} has right half plane zeros, then cancellation of the common zero may cause negative coefficients to appear in the numerator polynomial of T'. If this is the case, then the common factor cannot be cancelled and nothing has been gained by producing the common zero. Of course, the possible appearance of negative coefficients can easily be checked before the actual zero-shifting is performed.

Some rules (easily derived using the root locus technique; see the Appendix) which assist one in determining just what admittance should be removed in order to shift a zero to S=a are:

(1) Partial removal of a pole (i.e. $g_i < G_i$) moves all zeros (except the zero at S=0, if it exists) toward the pole at $-b_i$.

(2) Partial removal of the capacitance C_∞ (i.e. $c_\infty < C_\infty$) shifts all zeros (same exception) toward $S=-\infty$. This is actually a special case of (1); the pole is at infinity.

(3) Partial removal of the conductance G_0 (i.e. $g_0 < G_0$) shifts all zeros toward the origin. Whereas complete removal (i.e. $g_0=G_0$) produces a zero at the origin. A zero at the origin can occur in y_{22}' only when y_{21} has a zero there. Otherwise, cancellation of S would not occur in T', and T' would not be realizable.

After the type of shunt removal has been decided upon, the coefficients of y can be calculated from Eqn. 2-1. To produce a zero in y_{22}' at S=a, choose y such that:

$$y(a) = y_{22}(a) . \qquad (2\text{-}7)$$

If the coefficients of y calculated from Eqn. 2-7 satisfy the restrictions given after Eqn. 2-6 (equality can hold only in the special cases mentioned above), then it is possible to use the proposed y to perfom the desired zero-shifting.

Series Removal. Another elementary removal from the network N is the series (Se) admittance y, removed as shown in Figure 2-3.

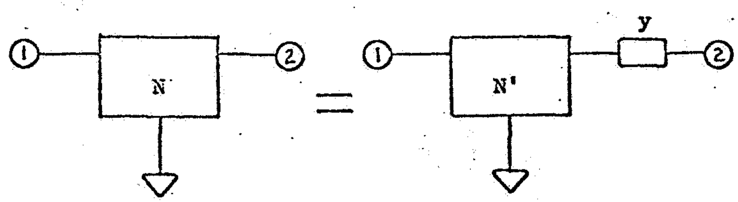


Figure 2-3. The series removal.

The parameters of the remaining network N' are easily calculated from:

$$\frac{1}{y_{22}'} = \frac{1}{y_{22}} - \frac{1}{y} , \tag{2-8}$$

$$T' = T , \tag{2-9}$$

and

$$-y_{21}' = T\, y_{22}' . \tag{2-10}$$

An RC driving point impedance (the reciprocal of Eqn. 1-7) can be expanded into partial fraction form with all non-negative coefficients. Thus, $1/y_{22}$ and $1/y$ can be written as:

$$\frac{1}{y_{22}} = R_{\infty} + \sum_i \frac{H_i}{S+a_i} \tag{2-11}$$

and

$$\frac{1}{y} = r_{\infty} + \sum_i \frac{h_i}{S+z_i} . \tag{2-12}$$

Subtracting Eqn. 2-12 from 2-11 gives $1/y_{22}'$. The partial fraction expansion of $1/y_{22}'$ must yield all non-negative coefficients in order for $1/y_{22}'$ to be an RC driving point impedance. Thus, each zero of y (pole of 1/y) must also be a zero of y_{22}. Then:

$$\frac{1}{y_{22}'} = (R_\infty - r_\infty) + \sum_i \frac{(H_i - h_i)}{S+a_i}, \qquad (2\text{-}13)$$

where: $r_\infty \leq R_\infty$ and $h_i \leq H_i$

are further restrictions which must be placed upon y.

It can be seen from Eqn. 2-13 that a zero of y_{22} need not be a zero of y_{22}'; the zero at $S=-a_i$, for example, can be eliminated by choosing $h_i=H_i$. By recalling that each pole of T' must be a zero of y_{22}' and by noting Eqn. 2-9, it can be concluded that complete removal of a zero from y_{22} is permissible only when the zero is a common zero. It is usually desirable to eliminate common zeros, since y_{22}' and y_{21}' are then less complex than y_{22} and y_{21}.

What else can be accomplished by a series removal? Equation 2-13 shows that the zeros of y_{22}' are all zeros of y_{22}, but that the poles of y_{22}' will in general be quite different from the poles of y_{22}. Hence, the series removal can be used as a pole-shifting step.

It is sometimes possible to use the pole-shifting property to simplify the realization of N. For example, assume that the poles of y_{22} are shifted in such a manner that y_{22}' has a pole where T has a zero (this is possible only when T has a negative zero). Now if T has no right half plane zeros, then cancellation can occur during the calculation of y_{21}' from Eqn. 2-10. The resulting N' is less complex than N since the order of y_{21}' is lower than the order of y_{21} (actually, a private pole has been produced). However, if y_{21} has right half plane zeros, then cancellation in Eqn. 2-10 may cause negative coefficients to appear in the numerator polynomial of $-y_{21}'$. If this is the case, then the factor cannot be cancelled and nothing has been accomplished by the pole shifting. Of course, the possible appearance of negative coefficients can easily be checked before the actual pole-shifting is performed.

Some rules (again easily derived using the root locus technique; see Appendix) which assist one in determining just what series admittance should be removed to shift a pole to S=a are:

(1) Partial removal of a zero (i.e. $h_i < H_i$) moves all finite poles toward the zero at $S=-a_i$.

(2) Partial removal of the series resistance R_∞ (i.e. $r_\infty < R_\infty$) shifts the poles toward $S=-\infty$. Complete removal (i.e. $r_\infty = R_\infty$) guarantees a pole in y_{22}' at infinity.

After deciding how to shift a pole to the desired position, the coefficients of the admittance y can be calculated from Eqn. 2-8. To produce in y_{22}' a pole at S=a by a series removal, choose y such that:

$$y(a) = y_{22}(a). \qquad (2\text{-}14)$$

If the coefficients of y calculated from Eqn. 2-14 satisfy the restrictions given after Eqn. 2-13 (h_i can equal H_i only when $-a_i$ is a common zero), then it is permissible to use the proposed y to perform the desired pole-shifting.

The Ladder Network. At this point it can be seen how the ladder class of networks can be developed. If only series and shunt removals are used during the development of a network, then the resulting network is a ladder. Shunt removals can be used to produce common zeros which can then be eliminated by series removals, and/or series removals can be used to produce private poles which can then be eliminated by shunt removals. T must have only real, non-positive zeros in order to completely develop a network with just series and shunt removals. This procedure is commonly called the 'zero-shifting' technique [18]. Example 2-1 gives a simple illustration of this realization procedure.

Example 2-1. Develop an RC ladder network having the transfer function T and the output admittance y_{22} given below.

$$y_{22} = \frac{(S+2)(S+4)}{S+3}, \quad T = K\frac{(S+1)(S+5)}{(S+2)(S+4)}, \Rightarrow -y_{21} = K\frac{(S+1)(S+5)}{S+3}.$$

A shunt capacitor can be used to produce a common zero at $S=-5$. Let $y=c_\infty S$. Eqn. 2-7 then gives the value of c_∞:

$$-5c_\infty = \frac{(2-5)(4-5)}{(3-5)} \Rightarrow c_\infty = 3/10.$$

Since C_∞ is equal to unity, the removal is permissible. The parameters of N' can be calculated from Eqns. 2-1, 2-2, & 2-3.

$$y_{22}' = \frac{(S+2)(S+4)}{S+3} - \frac{3}{10}S = \frac{7(S+16/7)(S+5)}{10(S+3)}.$$

$$y_{21}' = y_{21} \quad \text{and} \quad T' = \frac{10K}{7}\frac{S+1}{S+16/7}.$$

N' must now be developed. This is essentially a new problem; the primes will now be dropped from the above parameters.

A series $y = (S+5)/h_2$ can be used to eliminate the common zero at $S=-5$. For complete removal, h_2 must equal H_2. The residue, H_2, at the pole of $1/y_{22}$ can be calculated from Eqn. 2-11:

$$H_2 = \left.\frac{S+5}{y_{22}}\right|_{S=-5} = 20/19.$$

Then y_{22}' can be calculated from Eqn. 2-8:

$$\frac{1}{y_{22}'} = \frac{10}{7}\,\frac{S+3}{(S+16/7)(S+5)} - \frac{20}{19}\,\frac{1}{S+5} = \frac{50}{133}\,\frac{\cancel{(S+5)}}{(S+16/7)\cancel{(S+5)}}.$$

The network which must now be realized is:

$$y_{22} = \frac{133}{50}(S+16/7)\ , \quad -y_{21} = \frac{19K}{5}(S+1)\ , \text{ and } T = \frac{10K}{7}\,\frac{S+1}{S+16/7}\ .$$

A shunt removal $y = g_0 = \frac{133}{50}(\frac{16}{7} - 1) = 171/50$ will produce a common zero at $S=-1$. This leaves the network:

$$y_{22} = 133(S+1)/50\ , \quad -y_{21} = 19K(S+1)/5\ , \text{ and } T = 10K/7\ .$$

This final network is just the series admittance $133(S+1)/50$ (i.e. eliminate the common zero). For this final network, $T \equiv 1$, which gives $10K/7 = 1$ or $K = 7/10$. This development is tabulated in Figure 2-4, where the network schematic is also given. Note that the removal columns of the table provide the information necessary to draw the schematic.

The upper bound on K for the ladder class of networks is 8/10 [8][30] for the T in this example. However, the maximum K for an RC 3 T.N. is unity. A maximum K, 3 T.N. realization will be given in a later example (see Example 2-4).

y_{22}	$-y_{21}$	T	y	type	N
$\frac{(S+2)(S+4)}{S+3}$	$\frac{K(S+1)(S+5)}{S+3}$	$\frac{K(S+1)(S+5)}{(S+2)(S+4)}$	$\frac{3}{10}S$	Sh	
$\frac{7(S+16/7)(S+5)}{10(S+3)}$	$\Downarrow$	$\frac{10K}{7}\,\frac{S+1}{S+16/7}$	$\frac{19}{20}(S+5)$	Se	
$\frac{133}{50}(S+16/7)$	$\frac{19K}{5}(S+1)$	$\Downarrow$	$\frac{171}{50}$	Sh	
$\frac{133}{50}(S+1)$	$\Downarrow$	$\frac{10K}{7} = 1$	$\frac{133}{50}(S+1)$	Se	

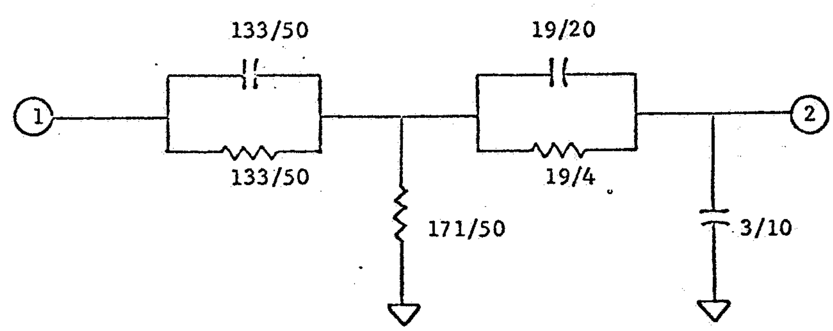


Figure 2-4. Tabulation of the development and the network schematic for Example 2-1.

The Network Gain Constant. Before extending the above technique, some comments about K are in order. The parameters T, y_{22}, and y_{21} for any network are unaltered whenever elements are connected between the input terminal and ground. Thus, the development of any network from these parameters essentially always terminates with a series removal. This termination of development can be brought about whenever the T to be realized is a constant - the product of a numerical constant and K. Since a network composed of a single series admittance has unity gain, K can be evaluated by setting the final constant expression for T equal to unity. This was illustrated in the preceding example. The usual method for calculating K (see for example: [2] [16] [20] [34] [36]), which in many cases involves complicated calculations, is actually unnecessary.

Suppose that a realization problem has been reduced to one where a series admittance removal will complete the realization. The parameters can then be written as:

$$y_{22} = y \ , \qquad -y_{21} = Ky/A \ , \qquad \text{and} \qquad T = K/A \ . \qquad (2\text{-}15)$$

If y is removed as a series admittance, then K = A for the network. It was implied earlier in this chapter that any K could then be realized, such that $0 < K \leq A$. This can be accomplished by first removing a shunt admittance (instead of the series y) equal to αy, where $0 \leq \alpha < 1$. The realization can then be completed with removal of a series admittance equal to $(1 - \alpha)y$. K for this realization can then be calculated from:

$$K/A(1-\alpha) = 1, \qquad \text{or:} \qquad K = A(1-\alpha) \leq A. \qquad (2\text{-}16)$$

Thus, any K such that $0 < K \leq A$ can be attained by properly choosing α. Given A from Eqn. 2-15 and the desired K, choose:

$$\alpha = 1 - \frac{K}{A}, \qquad (2\text{-}17)$$

and then perform the above mentioned shunt and series removals to complete the realization.

2.2 The Bridge Removal.

If T has one or more pairs of complex zeros, then eventually the pole-zero shifting technique will fail to simplify the remaining network. Consequently, the network development cannot be completed. To enable realization of more general types of T, another type of removal is needed. In an attempt to fulfill this need a new type of removal is proposed - a removal that can be used to shift the zeros of y_{21}. If the complex zeros of y_{21} can be shifted to the real axis, then the network development can be completed via the common zero - private pole technique. To accomplish the shifting of the zeros of y_{21} the bridge (Br) removal has been created. It consists of removing an admittance Ky from $-y_{21}$ and an admittance y from y_{22} as shown in Figure 2-5.

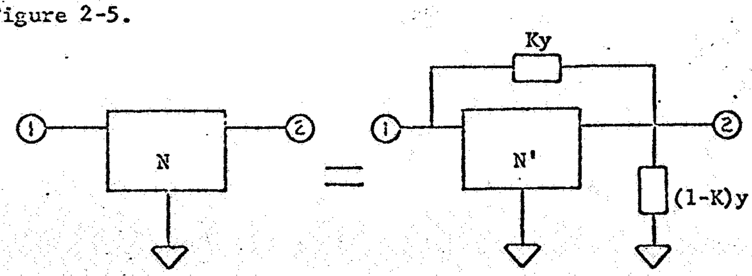


Figure 2-5. The bridge removal.

Even though the exact element values of the admittances in Figure 2-5 cannot be determined until after K is calculated at the completion of the realization, the bridge removal has been constructed to enable immediate calculation of the parameters of N':

$$y_{22}' = y_{22} - y , \qquad (2\text{-}18)$$

$$-y_{21}' = -y_{21} - Ky , \qquad (2\text{-}19)$$

and

$$T' = -y_{21}'/y_{22}' . \qquad (2\text{-}20)$$

Note that the factor K appears in the removal from $-y_{21}$. Since $-y_{21}$ itself contains the factor K, the zero shifting can be performed without actually knowing K. However, the total admittance removed from y_{22} contains no such factor K which enables immediate calculation of y_{22}'. This feature was provided by the shunt admittance (1-K)y.

The admittance y in Eqn. 2-18 must meet the same requirements that were imposed on y for the shunt removal since y_{22}' is calculated in exactly the same manner. A pole should not be completely removed from y_{22} unless the same pole is simultaneously removed from y_{21}. Zero shifting must not produce a zero in y_{22} at S=0 unless a zero is simultaneously produced in y_{21} at S=0 by the Ky subtraction. The admittance y must also be chosen so that the numerator polynomial of $-y_{21}'$ will not have any negative coefficients. If a negative coefficient appears, then N' could not be realized by an RC 3 T.N.

The restrictions which must be placed upon the general bridge removal can be summarized as:

(1) y must satisfy the shunt removal restrictions.

(2) All coefficients of the numerator polynomial of $-y_{21}'$ must be non-negative.

If a proposed y allows these two conditions to be satisfied, then the bridge removal can be performed. Two specific types of bridge removal will be discussed in detail later in this section.

<u>Some Necessary Precautions</u>. Building the network in Figure 2-5 will in general require realization of the admittance $(1-K)y$. Therefore, whenever the bridge removal is employed during the development of a network, special precautions are necessary to avoid the appearance of negative elements (i.e. negative elements may occur if K is permitted to exceed unity). The author suggests the following approach, especially for the novice. At the outstart, T can be written (scaled as noted after Eqn. 1-19) in such a manner that the upper bound on K is unity. The simplest way to accomplish this is to multiply T (and consequently $-y_{21}$) by the constant: $\min_{\forall i} (d_i/c_i)$ where d_i and c_i are the coefficients of the given T per Eqn. 1-19. Then application of Eqn. 1-22 guarantees that the K of any RC 3 T.N. realization of the resulting T will satisfy the inequality $K \leq 1$.

(Note that whenever bridge removals are employed, a K of exactly unity will in many cases require a fewer number of elements since the admittance $(1-K)y$ disappears. Thus, the above scaling of the maximum possible K causes the two desirable features, realization of maximum K and realization of unity K, to occur simultaneously.)

Unfortunately, after a bridge removal has been made, the maximum possible K for the remaining network N' may exceed the maximum K of unity which was set up at the outstart of the development of N (such a situation arises in Example 2-3). This presents no serious obstacle, however, since a gain reduction step can always be used at the completion of the development of N" to prevent K from exceeding unity. As a matter of fact, the gain reduction step can be used to obtain a K of exactly unity - the result being a network with maximum possible gain. If N' were mistakenly developed with a K that exceeds unity, then realization of the admittances dependent upon K would require use of at least one negative element, and the overall network would not be an RC 3 T.N.

As an alternative to the above approach, the reader may wish to experiment with scaling the maximum possible K to some number other than unity. In such cases the realization should be carried out only to the point where a final series removal will complete the development. At this point, element values which are dependent upon K can be examined. If completion of the development by a series removal would introduce no negative elements, then it should be performed. If the series removal would cause a K sufficiently large to introduce negative elements, then a gain reduction step should be employed to reduce K just enough to eliminate all negative elements.

The Bridge Conductance Removal. One of the most useful types of bridge removal is the bridge conductance, $y = g_0$. This removal can sometimes be used to produce a zero in y_{21}' at $S = a$. The value of the removal can be calculated by evaluating Eqn. 2-19 at $S = a$:

$$g_0 = \frac{-y_{21}(a)}{K} , \qquad (2\text{-}21)$$

where it is required that:

(1) $0 \leq g_0 \leq G_0 = y_{22}(0)$, where $g_0 = G_0$ is permissible only when $a = 0$.

(2) $-y_{21}'$ must have all non-negative polynomial coefficients.

If the proposed removal meets these restrictions, then it can be performed with assurance that N' will be a realizable RC 3 T.N. A common zero might then be produced by a shunt removal and subsequently eliminated by a series removal. This procedure will be illustrated in Example 2-2.

As mentioned in Section 2.1, production of a common zero requires special attention whenever the transfer functions have right half plane zeros. Production of a common zero at $S = a$ accomplishes nothing if cancellation of the factor $(S-a)$ from the numerator of $-y_{21}'$ leaves a polynomial with negative coefficients. If $a = 0$, however, restriction (2) above guarantees that no negative coefficients will appear. Hence, a common zero can be produced at $S = 0$, and can subsequently be eliminated by a series removal. This fact makes production of a zero in y_{21}' at $S = 0$ an especially desirable purpose for the bridge conductance removal.

Example 2-2. Develop a network having the following parameters:

$$y_{22} = \frac{(S+1)(S+3)}{S+2} \quad \text{and} \quad T = K\,\frac{S^2+2S+2}{(S+1)(S+3)} .$$

Tabulation of the development and the network schematic are given in Figure 2-6. Note that Br is used to indicate a bridge type removal. A bridge conductance is removed first to produce a zero in y_{21}' at S=0. The value of the removal is calculated from Eqn. 2-21: $g_0 = -y_{21}(0)/K = 1$. $y_{22}(0) = 3/2$ and $(S^2+2S+2) - (S+2) = S^2+S$, which show that restrictions (1) and (2) above are satisfied. Thus, the removal can be performed.

The remaining network can be realized as a ladder since the zeros of T are both real. The particular realization given in Figure 2-6 was accomplished by producing a common zero at S=0 through removal of a shunt conductance. A series capacitor was then used to eliminate this common zero. Another shunt conductance removal produced a common zero at S=-1. Elimination of this common zero was accomplished by the final series removal.

The Bridge Capacitance Removal. Another basic and useful removal is the bridge capacitance, $y = c_\infty S$. The bridge removal of a capacitance can sometimes be used to produce a zero in y_{21}' at S = a. The value of the removal can be calculated by dividing Eqn. 2-19 by KS and evaluating the result at S = a; this gives:

y_{22}	$-y_{21}$	T	y	type	N
$\frac{(S+1)(S+3)}{S+2}$	$K\ \frac{S^2+2S+2}{S+2}$	$K\ \frac{S^2+2S+2}{(S+1)(S+3)}$	1	Br	
$\frac{S^2+3S+1}{S+2}$	$K\ \frac{S(S+1)}{S+2}$	$K\ \frac{S(S+1)}{S^2+3S+1}$	$\frac{1}{2}$	Sh	
$\frac{S(S+5/2)}{S+2}$	$\Downarrow$	$K\ \frac{S+1}{S+5/2}$	$\frac{5}{2}\ S$	Se	
$5(S+5/2)$	$5K\ \ (S+1)$	$\Downarrow$	$\frac{15}{2}$	Sh	
$5(S+1)$	$\Downarrow$	$K = 1$	$5(S+1)$	Se	

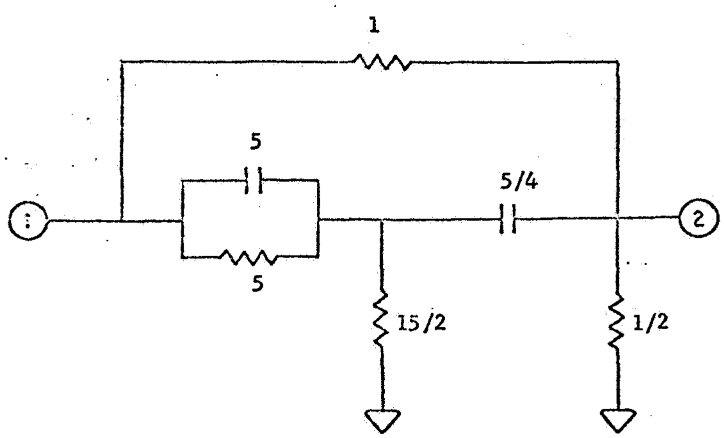


Figure 2-6. Tabulation of the development and the network schematic for Example 2-2.

$$c_{\infty} = \frac{-y_{21}(a)}{aK}, \tag{2-22}$$

where it is required that:

(1) $0 \leq c_{\infty} \leq C_{\infty} = \left.\frac{y_{22}}{S}\right|_{S\to\infty}$, where $c_{\infty} = C_{\infty}$ is permissible only when $a = \infty$.

(2) $-y_{21}'$ must have all non-negative polynomial coefficients. Note that requirements (1) and (2) cannot be met unless y_{22} and y_{21} both have poles at infinity.

If the proposed removal meets these restrictions, then it can be performed with assurance that N' will be a realizable RC 3 T.N. A common zero at S = a might then be produced, and subsequently eliminated by a series removal. Again, however, common zeros are advantageous only when their cancellation leaves a polynomial with all non-negative coefficients in the numerator of $-y_{21}'$.

The special, and perhaps most useful, purpose for this type of removal is elimination of the pole of $-y_{21}$ at infinity. This can be interpreted as shifting a zero to infinity (i.e. $a = \infty$) to cancel the pole there. Furthermore, if y_{22}' has a pole at infinity, it is then a private pole which can be eliminated by a shunt capacitance removal in the next step of the development.

Example 2-3 develops other networks for the parameters given in Example 2-2 to illustrate use of the bridge capacitor removal.

Example 2-3. Develop a network having the same parameters specified in Example 2-2.

This example will be worked twice to illustrate two different attacks to the gain reduction problem which arises in the development after the bridge capacitance removal. The two developments and schematics are shown in Figures 2-7 and 2-8. In both developments a bridge capacitance is removed first to eliminate the pole of $-y_{21}$ at infinity. The value of the capacitor is calculated from Eqn. 2-22 with $a = \infty$. The remaining network, after this removal, can be realized as a low pass ladder.

The first solution (Figure 2-7) realizes this ladder by producing a private pole at infinity with a series (y=2) removal. This pole is then eliminated with a shunt capacitance removal (y=4S). If the development were completed at this point with a series removal, then (see line four of the table) K would equal 3/2. Because of the bridge removal used earlier, a K exceeding unity would require a negative capacitor. Thus, the gain reduction procedure must be applied with A = 3/2. With K chosen to be unity, which is the maximum possible K, Eqn. 2-17 gives $\alpha = 1/3$. Hence, a shunt removal of $y = 6\alpha = 2$ is used to reduce the gain. The final removal is then just a series conductance.

The second solution (Figure 2-8) reduces the gain of the low pass network at the outstart of its development. This is done by providing a d.c. path to ground. This method has the advantage of providing a load resistance at the output terminals, which is desirable in many applications.

Since the d.c. gain of the low pass network determines K, the amount of conductance g_0 can be calculated to yield a K

y_{22}	$-y_{21}$	T	y	type	N
$\frac{(S+1)(S+3)}{S+2}$	$K\ \frac{S^2+2S+2}{S+2}$	$K\ \frac{S^2+2S+2}{(S+1)(S+3)}$	S	Br	
$\frac{2(S+3/2)}{S+2}$	$K\ \frac{2}{S+2}$	$K\ \frac{1}{S+3/2}$	$\frac{1}{2}$	Sh	
$\frac{3}{2}\ \frac{S+4/3}{S+2}$	$\Downarrow$	$\frac{4K}{3}\ \frac{1}{S+4/3}$	$\frac{3}{2}$	Se	
$\frac{9}{4}\ (S+4/3)$	$3K$	$\Downarrow$	$\frac{9}{4}\ S$	Sh	
3	$\Downarrow$	$K = 1$	3	Se	

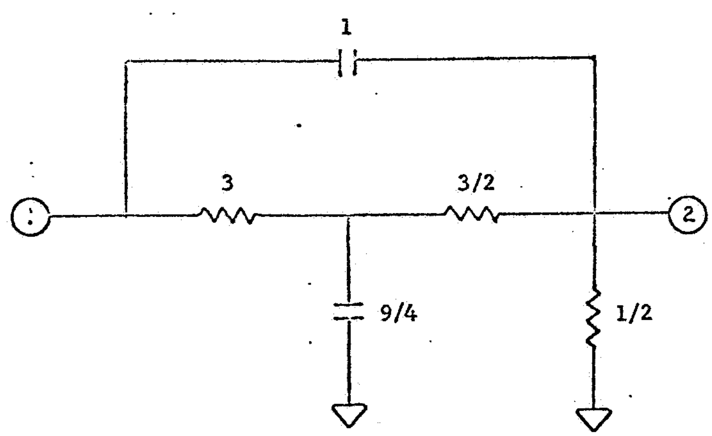


Figure 2-8. Tabulation of the development and the network schematic for Example 2-3, second realization.

of exactly unity. If, in the development of the remaining network, care is taken not to have any d.c. paths to ground (other than g_0 at the output), then the d.c. gain will be:

$$T(0) = \frac{y_{22}(0) - g_0}{y_{22}(0)} = 1 - \frac{g_0}{y_{22}(0)} ,$$

where the parameters here specify the network after the bridge removal (see line two of the table). Solving this equation for g_0 gives: $g_0 = y_{22}(0) \left[1 - T(0)\right]$. Letting K be unity gives $T(0) = 2/3$; thus, $g_0 = 1/2$.

As expected, after this shunt removal (line three of table), the maximum K for the remaining network is unity. The realization of this network is carried out without any d.c. paths to ground. Consequently, the realized K is unity.

Additional Comments About Bridge Removals. The bridge removal is not only useful to shift complex zeros to the real axis (or infinity) where they can then be realized by shunt-series removals. It can be applied to network functions which are already realizable by a ladder type network. In many cases this will permit a higher network gain than that which could be attained with a ladder structure. To illustrate this, Example 2-4 develops a network for the parameters specified in Example 2-1. A K of unity is obtained by using a bridge removal, whereas the upper bound on K for the ladder structure is 8/10.

Example 2-4. Develop a network with the parameters T and y_{22} specified in Example 2-1.

Tabulation of the development and the network schematic

y_{22}	$-y_{21}$	T	y	type	N
$\frac{(S+2)(S+4)}{S+3}$	$K \frac{(S+1)(S+5)}{S+3}$	$K \frac{(S+1)(S+5)}{(S+2)(S+4)}$	S	Br	
$\frac{3(S+8/3)}{S+3}$	$3K \frac{S+5/3}{S+3}$	$K \frac{S+5/3}{S+8/3}$	3	Se	
$9(S+8/3)$	$9K\ (S+5/3)$	$\Downarrow$	9	Sh	
$9(S+5/3)$	$\Downarrow$	K = 1	9S+15	Se	

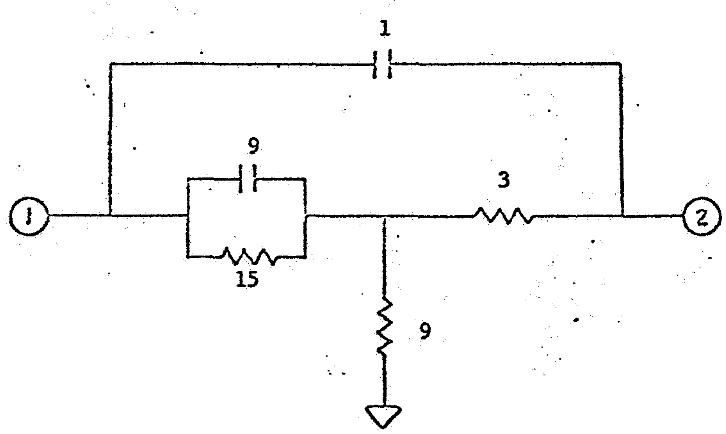


Figure 2-9. Tabulation of the development and the network schematic for Example 2-4.

are given in Figure 2-9. The development is very straightforward and needs no further explanation.

Unfortunately, the bridge removal does not always simplify realization in the straightforward manner that it did in Examples 2-2 and 2-3. It is easy to construct a problem where the transfer function has a pair of complex zeros yet a bridge removal does not simplify the network realization. Such a problem is given in the next example.

Example 2-5. Develop a network having the following network parameters:

$$y_{22} = \frac{(S+1)(S+3)}{S+2} \quad \text{and} \quad -y_{21} = K\,\frac{S^2+S+3}{S+2}\,.$$

A bridge removal in this case fails to improve the set of parameters; that is, fails to make the remaining network easier to realize. If, however, the pole of y_{21} were at or closer to the origin than S=-1 (at or further from the origin than S=-3), then a bridge capacitance removal (a bridge conductance removal) would simplify the realization problem.

Since a series removal can be used to shift the poles of y_{22} and hence the poles of y_{21}, the first approach that might be tried is to shift the pole of y_{22} to a region where the bridge removal will be of some use. However, the series removal permits shifting the pole only to values between the two zeros of y_{22}. It is desired to shift the pole outside of this region. The difficulty can be overcome by first using a shunt removal to shift the zeros of y_{22}.

y_{22}	$-y_{21}$	T	y	type	N
$\frac{(S+1)(S+3)}{S+2}$	$K\ \frac{S^2+S+3}{S+2}$	$K\ \frac{S^2+S+3}{S^2+4S+3}$	$\frac{9}{20}$	Sh	
$\frac{(S+3/4)(S+14/5)}{S+2}$	0	$\frac{K(S^2+S+3)}{(S+3/4)(S+14/5)}$	$\frac{9}{5}(S+3/4)$	Se	
$\frac{9(S+3/4)(S+14/5)}{4(S+1)}$	$\frac{9K}{4}\ \frac{S^2+S+3}{S+1}$	0	$\frac{9}{4}\ S$	Br	
$\frac{459}{80}\ \frac{S+14/17}{S+1}$	$\frac{27K}{4}\ \frac{1}{S+1}$	$\frac{20K}{17}\ \frac{1}{S+14/17}$	$\frac{459}{80}$	Se	
$\frac{2601}{80}\ (S+14/17)$	$\frac{153}{4}\ K$	0	$\frac{2601}{80}\ S$	Sh	
$\frac{1071}{40}$	0	$\frac{10K}{7} = 1$	$\frac{1071}{40}$	Se	

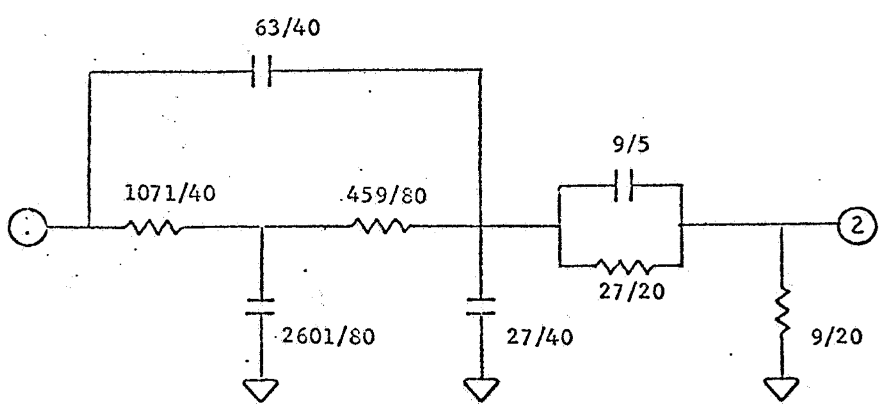


Figure 2-10. Tabulation of the development and the network schematic for Example 2-5.

A shunt removal is therefore used first in the development. The value is calculated to shift a zero to S=-3/4. The resulting network has a maximum K of only 7/10. If the zero were shifted to a value closer to -1, then a higher gain could be achieved.

A series removal is then used to shift the pole of y_{22} to S=-1. With the pole in this position, the bridge removal can be effectively used. The complete development is tabulated in Figure 2-10.

2.3 The Parallel Network Removal.

In order to guarantee generality of the realization procedure and to provide alternate solutions for the 'more difficult to realize' network parameters, a fourth type of removal will be added to the network development procedure - the parallel network (P) removal. Splitting a network into two (or more) parallel networks in order to simplify realization is by no means a new idea. It has been employed by Guillemin [16], Ozaki [28], and Fialkow and Gerst [9], just to name a few.

The parallel network removal will be presented here as an extension of the bridge removal. With a bridge removal, only RC driving point admittances can be subtracted from $-y_{21}/K$ in an attempt to shift the complex zeros of y_{21} to better positions. In many cases it is more desirable to subtract functions which are not RC driving point admittances. For instance, given the parameters of Example 2-5, it would be convenient if $KS^2/(S+2)$ or $3K/(S+2)$ could be subtracted from $-y_{21}$. The parallel network removal permits either type of subtraction.

A general parallel network removal is shown in Figure 2-11.

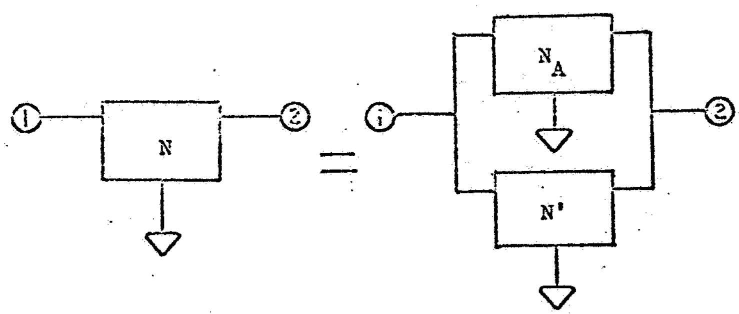


Figure 2-11. The parallel network removal.

The parameters of N_A will be called y_{22A}, y_{21A}, and T_A. The parameters of N' can be calculated from:

$$y_{22}' = y_{22} - y_{22A} , \quad (2\text{-}23)$$

$$-y_{21}' = -y_{21} + y_{21A} , \quad (2\text{-}24)$$

$$\text{and} \quad T' = -y_{21}'/y_{22}' . \quad (2\text{-}25)$$

Note that there are actually two remaining networks to be realized: N_A and N'. Each of these will be realized to within a constant multiplier K. This may require use of a gain reduction step in the development of one (or more in case several P removals are used) of the networks in order to obtain the same K for each network. Therefore, it is suggested that each of these networks be developed only up to the point where a series removal will complete their realizations. At this point the maximum K is easily determined for each network. The minimum of these values can then be chosen, and each network development can be completed in such a manner that its K is exactly the above value.

Tabulation of $\mathcal{P}$ Removals. In the development table, whenever a parallel network is removed, the symbol $\mathcal{P}$ should be entered in the 'type' of removal column. In the 'N' column a letter, such as A, should be entered to designate that the network N_A has been removed from the network being developed. Additional letters should be used in the event that other parallel networks are removed during the development of N' (e.g. B, C, D, etc.).

At the time of the removal of N_A, the parameters of N_A should be entered in another table or sufficiently far down in the same table. Throughout the development of N_A the letter 'A' should be placed in the network designation (N) column. If it is necessary to remove parallel networks during the development of N_A, the designations AA, AB, AC, etc. should be used.

Thus, a letter or letter group appears in the development table for the first time when it is removed from the network which is being developed at that time in the table. The next time that this letter or letter group appears in the table is when this particular network is to be developed. It then continues to appear until the development of this particular network is completed. Future examples will help to clarify use of the 'N' column.

Only two specific parallel network removals will be considered in detail - the strict high pass ladder and the strict low pass ladder. In either case, specification of y_{22A} completely determines the characteristics of the removal. Thus, y_{22A} will be entered in the y removal column together with an H (high pass) or L (low pass). This information is all that is needed to develop N_A at a later time.

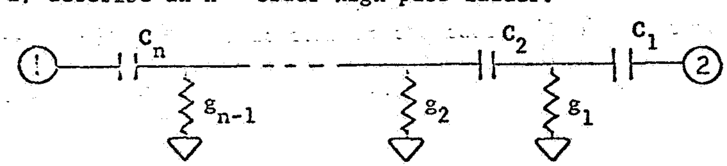
The High Pass Ladder Removal. Cauer's well known continued fraction expansion of an RC driving point admittance (e.g. y_{22}) about the origin gives a ladder network with all transmission zeros (i.e. zeros of T) at the origin (see Figure 2-12). If such a network is considered as a parallel removal, N_A, then:

$$y_{22A} = c_\infty S + \sum_{i=1}^{n-1} \frac{a_i S}{S+p_i}, \qquad (2\text{-}26)$$

and:

$$-y_{21A} = c_\infty K S^n \prod_{i=1}^{n-1} \frac{1}{S+p_i}. \qquad (2\text{-}27)$$

where $K = 1$ unless some capacitive path to ground is permitted in the development, in which case $K<1$. High pass ladders will later be referred to as first order, second order, etc. Equations 2-26 and 2-27 describe an n^{th} order high pass ladder.



$$y_{22A} = \cfrac{1}{\cfrac{1}{SC_1} + \cfrac{1}{g_1 + \cfrac{1}{\cfrac{1}{SC_2} + \dots + \cfrac{1}{\cfrac{1}{SC_n}}}}}.$$

Figure 2-12. An n^{th} order high pass ladder, N_A.

Equation 2-23 shows that y_{22}' is calculated in the same manner as y_{22}' for a shunt removal. Thus, y_{22A} must satisfy the restrictions placed upon y in the shunt removal discussion of Section 2.1. Furthermore, $g_0 = 0$; and a pole of y_{22} can be completely removed only when the corresponding pole of $-y_{21}$ is simultaneously removed. The coefficients of y_{22A} in Eqn. 2-26 are otherwise arbitrary. They might be selected to perform some zero shifting as an extra feature of the parallel network removal.

<u>Example 2-6</u>. Develop a network for the parameters given in Example 2-5.

Figure 2-13 gives the table and schematic for this development. A second order high pass ladder, N_A, is removed from N at the outstart. The pole at infinity is eliminated from both y_{22}' and y_{21}'. The residue of y_{22A}/S at S=-2 was arbitrarily chosen to be 1/4. Thus:

$$y_{22A} = S + \frac{1}{4}\frac{S}{S+2} = \frac{S(S+9/4)}{S+2},$$

which is the expression recorded in the y column for this removal.

<u>The Low Pass Ladder Removal</u>. Cauer's continued fraction expansion of an RC driving point admittance (e.g. y_{22}) about infinity gives a ladder network with all transmission zeros at infinity (see Figure 2-14). If such a network is considered as a parallel removal, then:

$$y_{22A} = g_0 + \sum_{1}^{n} \frac{a_i S}{S+p_i}, \qquad (2\text{-}28)$$

y_{22}	$-y_{21}$	T	y	type	N
$\frac{(S+1)(S+3)}{S+2}$	$K\ \frac{S^2+S+3}{S+2}$	$K\ \frac{S^2+S+3}{S^2+4S+3}$	$\frac{S(S+9/4)}{S+2}^{H}$	P	A
$\frac{7}{4}\ \frac{S+12/7}{S+2}$	$K\ \frac{S+3}{S+2}$	$\frac{4K}{7}\ \frac{S+3}{S+12/7}$	$\frac{7}{4}$	Se	
$\frac{49}{8}(S+12/7)$	$\frac{7K}{2}(S+3)$	$\Downarrow$	$\frac{21}{8}\ S$	Sh	
$\frac{7}{2}(S+3)$	$\Downarrow$	$K = 1$	$\frac{7}{2}(S+3)$	Se	
$\frac{S(S+9/4)}{S+2}$	$\frac{KS^2}{S+2}$	$\frac{KS}{S+9/4}$	$\frac{9}{8}\ S$	Se	A
$9(S+9/4)$	$9KS$	$\Downarrow$	$\frac{81}{4}$	Sh	A
$9S$	$\Downarrow$	$K = 1$	$9\ S$	Se	A

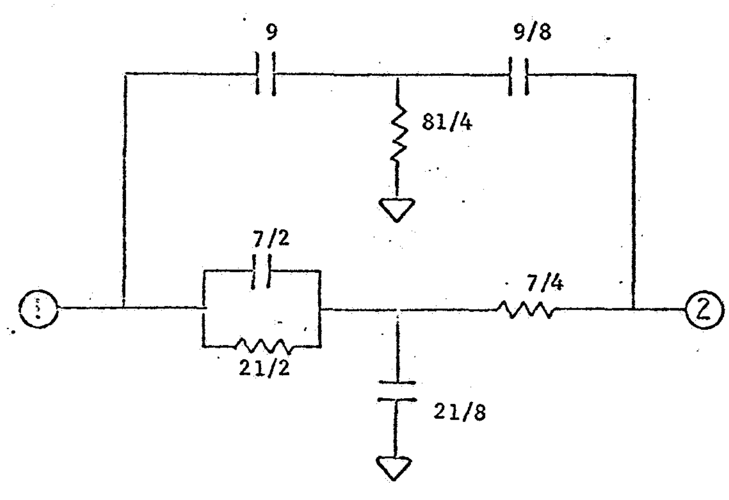


Figure 2-13. Tabulation of the development and the network schematic for Example 2-6.

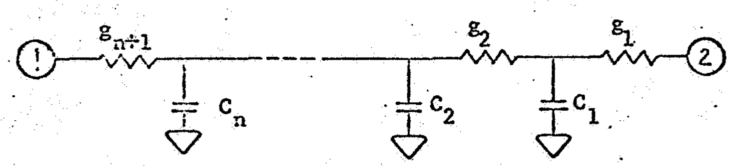
$$\text{and} \quad -y_{21A} = g_0 K \prod_{i=1}^{n} \frac{p_i}{S+p_i} , \qquad (2\text{-}29)$$

where K = 1 unless some resistive path to ground is permitted during the development, in which case $K<1$.

As in the high pass case, y_{22A} must satisfy the shunt removal restrictions. Furthermore, $c_\infty = 0$; a pole of y_{22} can be completely removed only when the corresponding pole of $-y_{21}$ is simultaneously removed; and, g_0 can equal $y_{22}(0)$ only if a zero is simultaneously produced in y_{21}' at S = 0. The coefficients of y_{22A} in Eqn. 2-28 are otherwise arbitrary.

Example 2-7. Develop a network having the parameters specified in Examples 2-5 and 2-6.

Figure 2-15 gives the table and schematic for this realization. A first order (n=1) low pass ladder, N_A, is removed from N at the outstart. The residue of y_{22A}/S at S=-2 was again arbitrarily chosen to be 1/4. g_0 was chosen to produce a zero in y_{21}' at S=0. The remainder of the development needs no explanation.



$$y_{22A} = \frac{1}{\frac{1}{g_1} + SC_1 + \frac{1}{\frac{1}{g_2} + \dots + \frac{1}{\frac{1}{g_{n+1}}}}} .$$

Figure 2-14. An n^{th} order low pass ladder, N_A.

y_{22}	$-y_{21}$	T	y	type	N
$\frac{(S+1)(S+3)}{S+2}$	$K\ \frac{S^2+S+3}{S+2}$	$K\ \frac{S^2+S+3}{S^2+4S+3}$	$\frac{7(S+12/7)}{4(S+2)}\ _L$	P	A
$\frac{S(S+9/4)}{S+2}$	$K\ \frac{S(S+1)}{S+2}$	$K\ \frac{S+1}{S+9/4}$	$\frac{9}{8}S$	Se	
$9(S+9/4)$	$9K(S+1)$	$\Downarrow$	$\frac{45}{4}$	Sh	
$9(S+1)$	$\Downarrow$	$K = 1$	$9(S+1)$	Se	
$\frac{7}{4}\ \frac{S+12/7}{S+2}$	$\frac{3K}{S+2}$	$\frac{12K}{7}\ \frac{1}{S+12/7}$	$\frac{7}{4}$	Se	A
$\frac{49}{8}(S+12/7)$	$\frac{21}{2}K$	$\Downarrow$	$\frac{49}{8}S$	Sh	A
$\frac{21}{2}$	$\Downarrow$	$K = 1$	$\frac{21}{2}$	Se	A

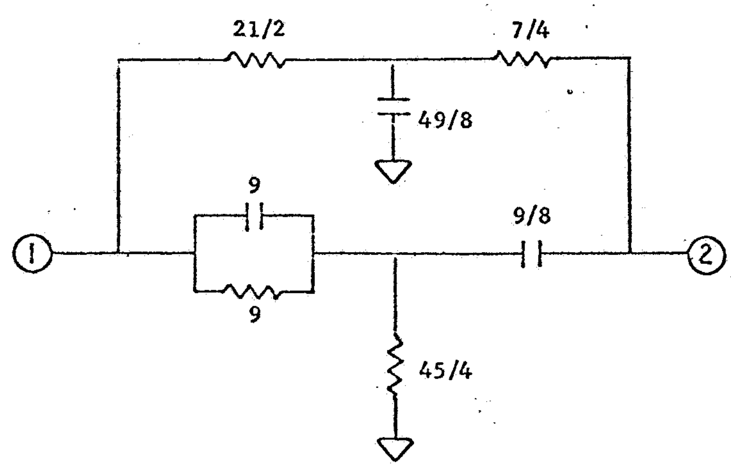


Figure 2-15. Tabulation of the development and the network schematic for Example 2-7.

Bridge Removal (Br)

Remaining network N':

$$\begin{aligned} y_{22}' &= y_{22} - y \\ -y_{21}' &= -y_{21} - Ky \\ T' &= -y_{21}'/y_{22}' \end{aligned}$$

Purpose: (1) To produce a real non-positive zero in y_{21}'.
(2) To eliminate the pole of y_{21} at infinity.

Special precaution: Either K must not exceed unity; or if $K > 1$, then it is necessary to reduce the gain to the extent that no negative elements appear in the network.

Parallel Network Removal (P)

Remaining network N':

$$\begin{aligned} y_{22}' &= y_{22} - y_{22A} \\ -y_{21}' &= -y_{21}' + y_{21A} \\ T' &= -y_{21}'/y_{22}' \end{aligned}$$

Purpose: (1) Same as for bridge removal. This removal has more flexibility, but usually requires more elements.

Special precaution: The K's for each network must be equal.

The author suggests using the following priority list when deciding upon what type of removal should be made (unless it is desired to have a particular network structure):

(1) Use a series removal whenever possible. Series removals never cause an immediate reduction of maximum possible gain.

(2) Use a shunt removal whenever possible, unless an undesirable reduction of gain is caused by the shunt removal.

(3) Use a bridge removal whenever series and shunt removals are impossible or undesirable, assuming of course that the

2.4 Summary and Examples.

This section presents a brief summary of the types of removals and their purposes. Simple examples have been given previously to illustrate certain items in the presentation of the realization procedure. Several additional examples are given in this section to show the ease with which slightly more complicated networks can be developed.

<u>Shunt Removal (Sh)</u>

Remaining network N':

$$y_{22}' = y_{22} - y$$
$$-y_{21}' = -y_{21}$$
$$T' = -y_{21}/y_{22}'$$

Purpose:
(1) To eliminate a private pole.
(2) To produce a common zero.
(3) To shift the zeros of y_{22}.

<u>Series Removal (Se)</u>

Remaining network N':

$$1/y_{22}' = 1/y_{22} - 1/y$$
$$T' = T$$
$$-y_{21}' = y_{22}'T$$

Purpose:
(1) To eliminate a common zero.
(2) To produce a private pole.
(3) To shift the poles of y_{22} and y_{21}.

bridge removal causes a simplification of the problem.

(4) Use a parallel network removal whenever the bridge removal is impossible (or possible only after excessive shifting of poles and zeros). If a high or low pass ladder is used, the order is an indication of the number of elements needed to realize the removal. Thus, it is desirable to remove the lowest order ladder which accomplishes simplification of the remaining network.

<u>Example 2-8</u>. The first example to be considered is given in Figure 2-16. The network is developed essentially by repetition of three steps: (1) removal of a bridge conductance to produce a zero of transmission at the origin; (2) removal of a shunt conductance to produce a common zero at the origin; and (3) removal of a series capacitance to eliminate the common zero. In this example it is not actually necessary to complete the calculation of y_{22}' after the bridge removal since the next removal is a shunt type. y_{22}' can be calculated after the shunt removal by summing the bridge and shunt conductances, and then subtracting the total from y_{22}. This saves some unnecessary calculations. Whenever a polynomial need not be calculated, 'NNC' will be entered in the table in place of the actual polynomial (unless the calculation is very simple, in which case the polynomial will be calculated).

<u>Example 2-9</u>. The second example to be considered (see Figure 2-17) makes use of a first order low pass ladder removal. A second order could have been used, but this would require more elements.

If only T is specified (not y_{22}), then a pole of y_{22} could be

y_{22}	$-y_{21}$	T
$\frac{S^4+19.000S^3+122.00S^2+296.00S+192.00}{(S+2.0000)(S+5.0000)(S+7.0000)}$	$K\ \frac{S^4+8.0000S^3+27.000S^2+38.000S+26.000}{(S+2.0000)(S+5.0000)(S+7.0000)}$	$\frac{K(S^2+2.0000S+2.0000)(S^2+6.0000S+13}{(S+1.0000)(S+4.0000)(S+6.0000)(S+8}$
$\frac{NNC}{S^3+14.000S^2+59.000S+70.000}$	$\frac{KS(S^3+7.6286S^2+21.800S+16.086)}{S^3+14.000S^2+59.000S+70.000}$	$\frac{KS(S^3+7.6286S^2+21.800S+16.086)}{NNC}$
$\frac{S(S^3+16.257S^2+83.600S+134.17)}{S^3+14.000S^2+59.000S+70.000}$	$\Downarrow$	$K\ \frac{S^3+7.6286S^2+21.800S+16.086}{S^3+16.257S^2+83.600S+134.17}$
$2.0908\ \frac{S^3+16.257S^2+83.600S+134.17}{S^2+11.538S+32.166}$	$2.0908K\ \frac{S^3+7.6286S^2+21.800S+16.086}{S^2+11.538S+32.166}$	$\Downarrow$
$2.0908\ \frac{NNC}{S^2+11.538S+32.166}$	$2.0908K\ \frac{S(S^2+7.1285S+16.030)}{S^2+11.538S+32.166}$	$K\ \frac{S(S^2+7.1285S+16.030)}{NNC}$
$2.0908\ \frac{S(S^2+12.086S+35.473)}{S^2+11.538S+32.166}$	$\Downarrow$	$K\ \frac{S^2+7.1285S+16.030}{S^2+12.086S+35.473}$
$22.428\ \frac{S^2+12.086S+35.473}{S+6.2073}$	$22.428K\ \frac{S^2+7.1285S+16.030}{S+6.2073}$	$\Downarrow$
$22.428\ \frac{NNC}{S+6.2073}$	$22.428K\ \frac{S(S+4.5460)}{S+6.2073}$	$K\ \frac{S(S+4.5460)}{NNC}$
$22.428\ \frac{S(S+6.3712)}{S+6.2073}$	$\Downarrow$	$K\ \frac{S+4.5460}{S+6.3712}$
$871.69\ (S+6.3712)$	$871.69K\ (S+4.5460)$	$\Downarrow$
$871.69\ (S+4.5460)$	$\Downarrow$	$K = 1$

Figure 2-16(a). Tabulation of the development for Example 2-8.

	$-y_{21}$	T	y	type	N
.00	$K\,\frac{S^4+8.0000S^3+27.000S^2+38.000S+26.000}{(S+2.0000)(S+5.0000)(S+7.0000)}$	$\frac{K(S^2+2.0000S+2.0000)(S^2+6.0000S+13.000)}{(S+1.0000)(S+4.0000)(S+6.0000)(S+8.0000)}$	0.37143	Br	
	$\frac{KS(S^3+7.6286S^2+21.800S+16.086)}{S^3+14.000S^2+59.000S+70.000}$	$\frac{KS(S^3+7.6286S^2+21.800S+16.086)}{NNC}$	2.3714	Sh	
	$\Downarrow$	$K\,\frac{S^3+7.6286S^2+21.800S+16.086}{S^3+16.257S^2+83.600S+134.17}$	1.9167 S	Se	
7	$2.0908K\,\frac{S^3+7.6286S^2+21.800S+16.086}{S^2+11.538S+32.166}$	$\Downarrow$	1.0456	Br	
	$2.0908K\,\frac{S(S^2+7.1285S+16.030)}{S^2+11.538S+32.166}$	$K\,\frac{S(S^2+7.1285S+16.030)}{NNC}$	7.6758	Sh	
	$\Downarrow$	$K\,\frac{S^2+7.1285S+16.030}{S^2+12.086S+35.473}$	2.3058 S	Se	
	$22.428K\,\frac{S^2+7.1285S+16.030}{S+6.2073}$	$\Downarrow$	57.918	Br	
	$22.428K\,\frac{S(S+4.5460)}{S+6.2073}$	$K\,\frac{S(S+4.5460)}{NNC}$	70.248	Sh	
	$\Downarrow$	$K\,\frac{S+4.5460}{S+6.3712}$	23.020 S	Se	
	$871.69K\ (S+4.5460)$	$\Downarrow$	1591.0	Sh	
	$\Downarrow$	$K = 1$	871.69S+3962.7	Se	

of the development for Example 2-8.

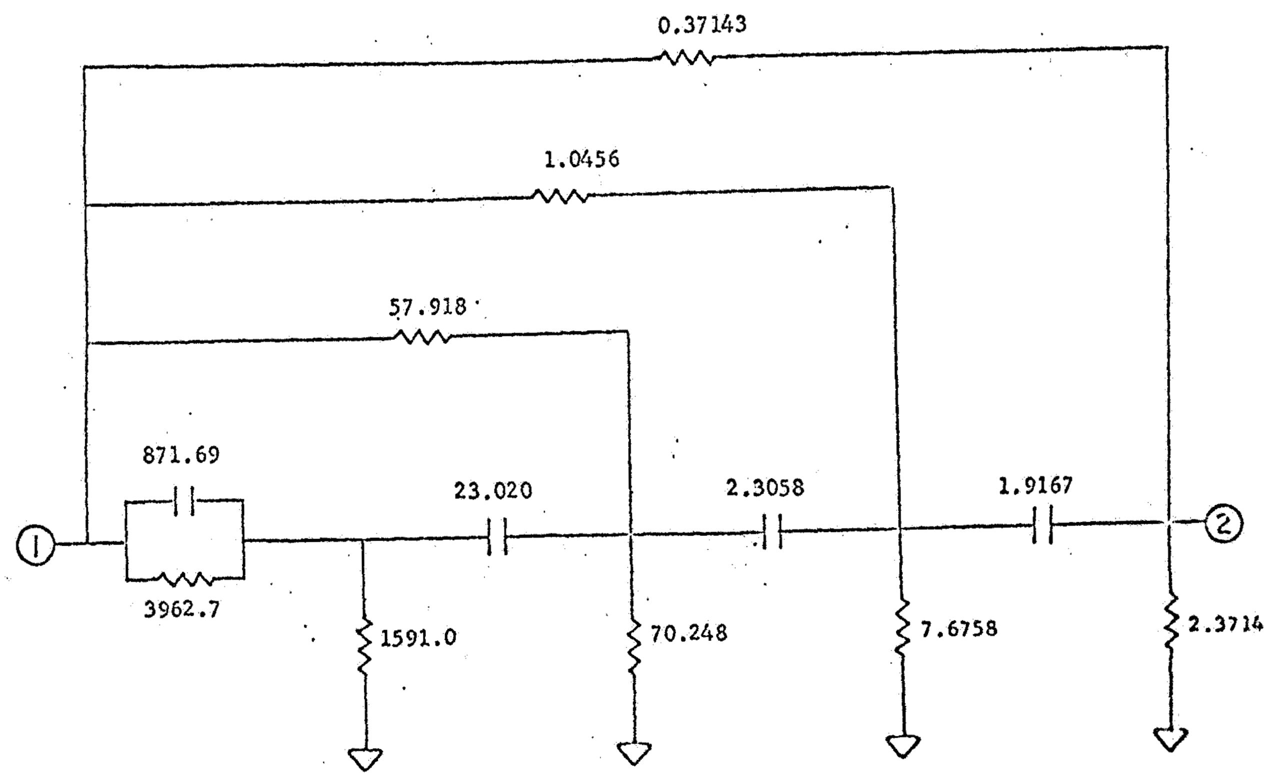


Figure 2-16(b). The network schematic for Example 2-8.

Network Parameters			Removal		
y_{22}	$-y_{21}$	T	y	type	N
$\frac{(S+2)(S+6)(S+10)}{(S+4)(S+8)}$	$\frac{K(S^3+S^2+4S+30)}{(S+4)(S+8)}$	$\frac{K(S+3)(S^2-2S+10)}{(S+2)(S+6)(S+10)}$	$\frac{15}{16}+\frac{\frac{1}{4}S}{S+4}$ H	P	A
$\frac{NNC}{(S+4)(S+8)}$	$\frac{KS(S^2+S+1/4)}{(S+4)(S+8)}$	$\frac{KS(S^2+S+1/4)}{NNC}$	$\frac{45}{16}$	Sh	
$\frac{S(S^2+55/4S+43)}{(S+4)(S+8)}$	$\downarrow$	$K\frac{S^2+S+1/4}{S^2+55/4S+43}$	$\frac{43}{32}S$	Se	
$\frac{43}{11}\frac{S^2+55/4S+43}{S+76/11}$	$\frac{43}{11}K\frac{S^2+S+1/4}{S+76/11}$	$\downarrow$	$\frac{43}{304}$	Br	
$\frac{43}{11}\frac{NNC}{S+76/11}$	$\frac{43KS(S+293/304)}{11(S+76/11)}$	$\frac{KS(S+293/304)}{NNC}$	$\frac{7353}{304}$	Sh	
$\frac{43}{11}\frac{S(S+143/19)}{S+76/11}$	$\downarrow$	$K\frac{S+293/304}{S+143/19}$	$\frac{6149}{1444}S$	Se	
$\frac{143}{3}(S+143/19)$	$\frac{143}{3}K(S+293/304)$	$\downarrow$	$\frac{95,095}{304}$	Sh	
$\frac{143}{3}(S+293/304)$	$\downarrow$	K = 1	143S/3 + 41899/912	Se	
$\frac{23}{16}\frac{S+60/23}{S+4}$	$\frac{15K}{4}\frac{1}{S+4}$	$\frac{60K}{23}\frac{1}{S+60/23}$	$\frac{23}{16}$	Se	A
$\frac{529}{512}(S+60/23)$	$\frac{345}{128}K$	$\downarrow$	$\frac{529}{512}S$	Sh	A
$\frac{345}{128}$	$\downarrow$	K = 1	$\frac{345}{128}$	Se	A

Figure 2-17(a). Tabulation of the development for Example 2-9, first problem.

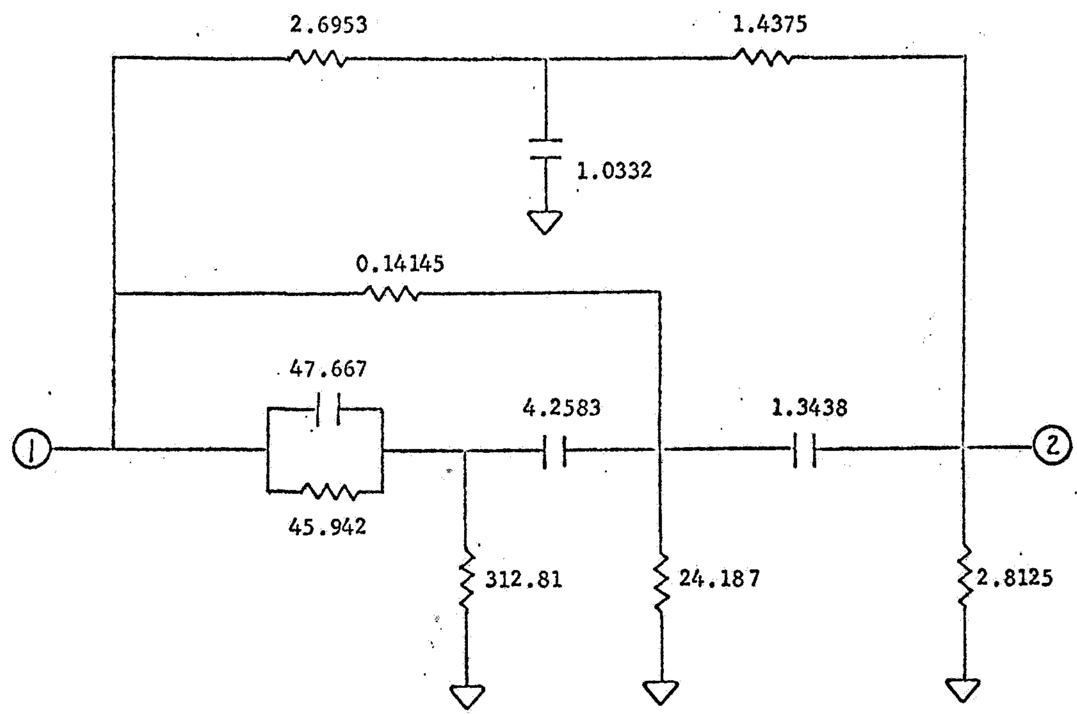


Figure 2-17(b). The network schematic for Example 2-9, first problem.

chosen at $S = -15/2$ instead of $S = -8$. This permits additional simplification by the first removal (a double zero at the origin is produced by the $\mathbb{P}$ removal), and consequently one less element is required for the overall network. Figure 2-18 gives the development for this choice of y_{22}.

Example 2-10. In the previous examples it was quite easy to develop maximum gain networks since the maximum occurred at infinite frequency. In this example, T has zeros at both $S = \infty$ and $S = 0$. Consequently, it is quite difficult to achieve a K near the upper bound of unity. For the development given in Figure 2-19, a K of only 0.273 was achieved.

The maximum realizable gain has been reduced twice during the development of the network. The first removal reduces it from unity to 9/10. If the poles of y_{22} had been chosen closer to $-\infty$ (e.g. poles at -2.5, -5.5, and -9.5), then less gain reduction would have occurred during this removal. However, the maximum K would be reduced to some number less than 19/20 no matter how close the poles of y_{22} are chosen to the zeros.

The next shunt removal (shunt capacitance) causes another reduction of realizable gain. After this removal (see line four of table), K cannot exceed 71/260. If the poles of y_{22} had been shifted closer to the origin prior to this shunt removal, then a less serious reduction in gain would have occurred. This could be accomplished by partial removal of the zero of y_{22} which is nearest the origin (this zero is located at approx. $S = -2.5594$). However, two

Network Parameters			Removal		
y_{22}	$-y_{21}$	T	y	type	N
$\frac{(S+2)(S+6)(S+10)}{(S+4)(S+15/2)}$	$\frac{K(S^3+S^2+4S+30)}{(S+4)(S+15/2)}$	$\frac{K(S+3)(S^2-2S+10)}{(S+2)(S+6)(S+10)}$	$1+\frac{S}{S+4}{}_L$	P	A
$\frac{(S+2)(S+5)(S+9)}{(S+4)(S+15/2)}$	$\frac{KS^2(S+1)}{(S+4)(S+15/2)}$	$\frac{KS^2(S+1)}{(S+2)(S+5)(S+9)}$	3	Sh	
$\frac{S(S^2+13S+77/2)}{(S+4)(S+15/2)}$	$\downarrow$	$\frac{KS(S+1)}{S^2+13S+77/2}$	$\frac{77}{60}S$	Se	
$\frac{77}{17}\frac{S^2+13S+77/2}{S+211/34}$	$\frac{77}{17}K\frac{S(S+1)}{S+211/34}$	$\downarrow$	$\frac{5929}{211}$	Sh	
$\frac{77}{17}\frac{S(S+1434/211)}{S+211/34}$	$\downarrow$	$\frac{K(S+1)}{S+1434/211}$	$\frac{220,836}{44,521}S$	Se	
$\frac{2868}{55}(S+1434/211)$	$\frac{2868}{55}K(S+1)$	$\downarrow$	$\frac{3,507,564}{11,605}$	Sh	
$\frac{2868}{55}(S+1)$	$\downarrow$	$K = 1$	$\frac{2868}{55}(S+1)$	Se	
$\frac{2(S+2)}{S+4}$	$\frac{4K}{S+4}$	$\frac{2K}{S+2}$	2	Se	A
$(S+2)$	$2K$	$\downarrow$	S	Sh	A
2	$\downarrow$	$K = 1$	2	Se	A

Figure 2-18(a). Tabulation of the development for Example 2-9, second problem.

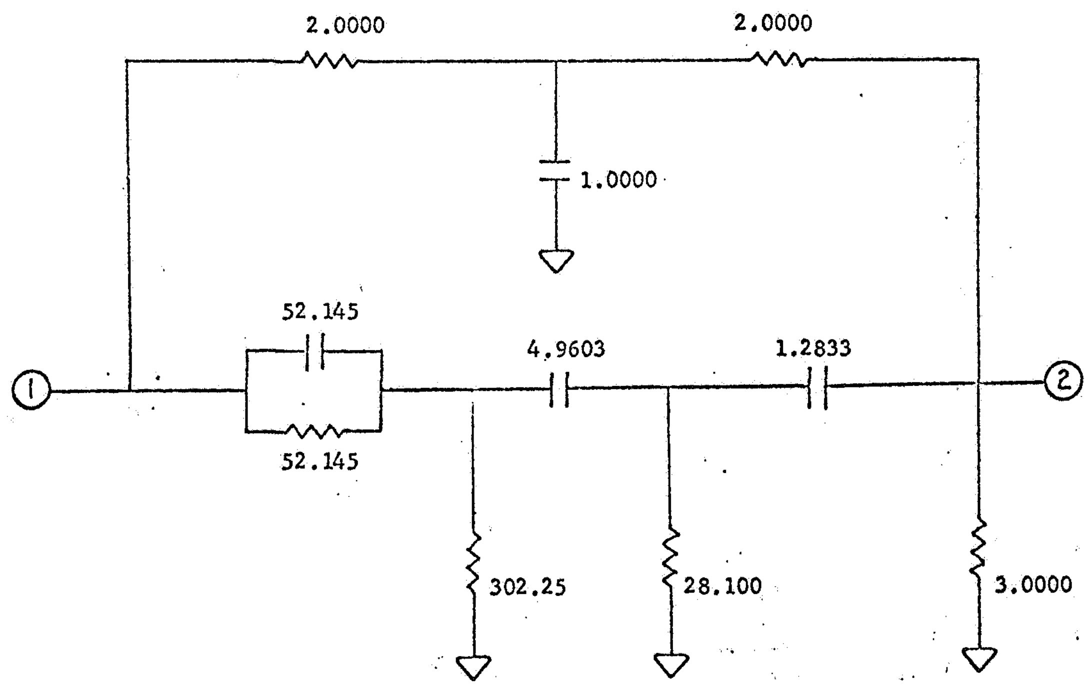


Figure 2-18(b). The network schematic for Example 2-9, second problem.

y_{22}	$-y_{21}$	T	y	type	N
$\frac{(S+1)(S+3)(S+6)(S+10)}{(S+2)(S+5)(S+9)}$	$\frac{20KS(S^2+S+7)}{(S+2)(S+5)(S+9)}$	$\frac{20KS(S^2+S+7)}{(S+1)(S+3)(S+6)(S+10)}$	2	Sh	
$\frac{S(S^3+18S^2+95S+142)}{(S+2)(S+5)(S+9)}$	$\Downarrow$	$\frac{20KS(S^2+S+7)}{S^3+18S^2+95S+142}$	$\frac{71}{45}S$	Sc	
$\frac{71}{2}\ \frac{S^3+18S^2+95S+142}{13S^2+163S+454}$	$\frac{710K(S^2+S+7)}{13S^2+163S+454}$	$\Downarrow$	$\frac{71}{26}S$	Sh	
$\frac{5041}{26}\ \frac{S^2+11S+26}{13S^2+163S+454}$	$\Downarrow$	$\frac{260K}{71}\ \frac{S^2+S+7}{S^2+11S+26}$	$\frac{5041}{338}$	Sc	
$\frac{5041}{520}\ \frac{S^2+11S+26}{S+29/5}$	$\frac{5041K}{142}\ \frac{S^2+S+7}{S+29/5}$	$\Downarrow$	$\frac{5041}{1248}(3+\sqrt{17})(S+\frac{11+\sqrt{17}}{2})$	Sc	
$\frac{5041}{52(19-3\sqrt{17})}\ \frac{S^2+11S+26}{S+7}$	$\frac{355K}{19-3\sqrt{17}}\ \frac{S^2+S+7}{S+7}$	$\Downarrow$	$\frac{355}{19-3\sqrt{17}}$	Br	
$\frac{5041}{52(19-3\sqrt{17})}\ \frac{NNC}{S+7}$	$\frac{355K}{19-3\sqrt{17}}\ \frac{S^2}{S+7}$	$\frac{260K}{71}\ \frac{S^2}{NNC}$	$\frac{71}{14(19-3\sqrt{17})}$	Sh	
$\frac{5041}{52(19-3\sqrt{17})}\ \frac{S(S+51/7)}{S+7}$	$\Downarrow$	$\frac{260K}{71}\ \frac{S}{S+51/7}$	$\frac{257,091}{2548(19-3\sqrt{17})}S$	Sc	
$\frac{257,091}{104(19-3\sqrt{17})}(S+51/7)$	$\frac{18,105}{2(19-3\sqrt{17})}KS$	$\Downarrow$	$\frac{13,111,641}{728(19-3\sqrt{17})}$	Sh	
$\frac{257,091}{104(19-3\sqrt{17})}S$	$\Downarrow$	260K/71 = 1 or: K = 0.273	$\frac{270,091}{104(19-3\sqrt{17})}S$	Sc	

Figure 2-19(a). Tabulation of the development for Example 2-10.

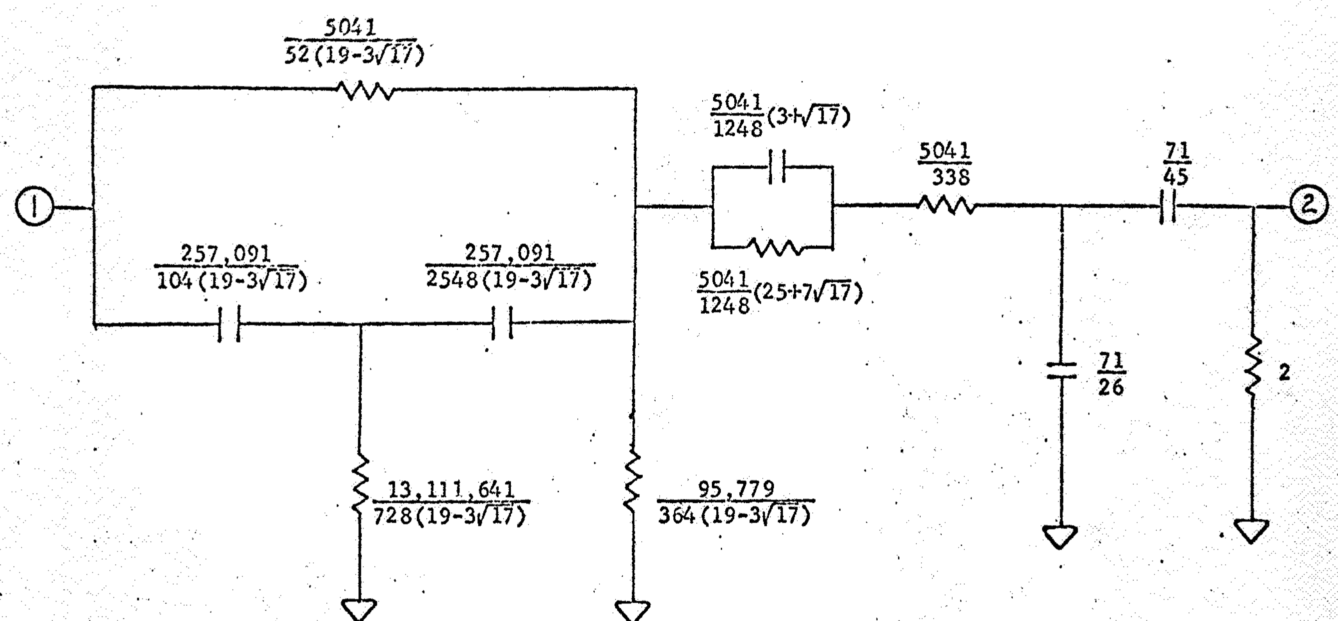


Figure 2-19(b). The network schematic for Example 2-10.

additional elements would be required.

No matter how much pole and zero shifting is done in an attempt to increase K, the basic approach of realizing the transmission zeros at $S=0$, $S=\infty$, and then the complex transmission zeros will never yield a K greater than or equal to 1/2. From this approach the resulting network is basically a cascade connection of three networks: one with complex transmission zeros, followed by the networks with transmission zeros at $S=\infty$, and $S=0$ respectively. If the loading of each cascaded network by the following one were negligible, then the three networks would have the transfer functions:

$\frac{S^2+S+7}{(S+3)(S+6)}$, $\frac{10}{S+10}$, and $\frac{S}{S+1}$ respectively. The product of these indicates a K of 1/2. However, K = 1/2 cannot be achieved exactly because the loading is never completely negligible.

In order to get a larger network gain it appears that a parallel network removal would have to be made (or perhaps a bridge removal made earlier in the development). In any event the basic cascade structure of the network in Figure 2-19 would have to be avoided. The use of a surplus factor such as (S+11) would probably make it much easier to realize a higher K.

Example 2-11. Realize the transfer function:

$$T = \frac{3K}{8} \frac{S^2+8}{(S+1)(S+3)} .$$

One approach to this problem might be to choose an appropriate pole for y_{22} and then begin the development. If this were done, it would be found that a bridge removal could not be used. Hence, either a low pass or a high pass parallel ladder would probably be the first removal. In either case the final result would be a seven element Twin - Tee type network.

However, if a surplus factor is introduced into T, then the parallel network type of removal can be avoided in the development. If the surplus factor is carefully chosen, then the number of elements remains the same. With a surplus factor introduced, the network parameters are:

$$T = \frac{3K}{8} \frac{(S^2+8)(S+a)}{(S+1)(S+3)(S+a)} ,$$

$$y_{22} = \frac{(S+1)(S+3)(S+a)}{(S+b)(S+c)} , \text{ and } -y_{21} = \frac{3K}{8} \frac{S^3+aS^2+8S+8a}{(S+b)(S+c)} .$$

Since one pole of y_{22} must lie between S=-1 and S=-3, let us choose b=2. If a bridge capacitance removal is to be used first to eliminate the pole of $-y_{21}$ at infinity, then it would be advantageous to have $-y_{21}'$ even further simplified during this removal (i.e. to have the first three coefficients drop out). Thus, it is desired to have $S^2+(b+c)S+bc$ equal S^2+aS+8. Equating coefficients gives c=4 and a=6. The complete network development is carried out in Figure 2-20.

The above procedure for selecting a surplus factor can also be applied successfully to the transfer function given in Examples 2-5, 2-6, and 2-7. If this is done, a six element realization can be obtained. The network structure would be the same as the structure of the network shown in Figure 2-20 except that no shunt capacitor would be connected to the output terminal.

y_{22}	$-y_{21}$	T	y	type	N
$\frac{(S+1)(S+3)(S+6)}{(S+2)(S+4)}$	$\frac{3K}{4}\,\frac{S^3+6S^2+8S+48}{S^2+6S+8}$	$\frac{3K(S^2+8)(S+6)}{8(S+1)(S+3)(S+6)}$	$\frac{3S}{8}$	Br	
$\frac{NNC}{(S+2)(S+4)}$	$18K\,\frac{1}{(S+2)(S+4)}$	$\frac{3K}{8}\,\frac{1}{NNC}$	$\frac{5S}{8}$	Sh	
$4\,\frac{S^2+19/4S+9/2}{(S+2)(S+4)}$	$\Downarrow$	$\frac{9K}{2}\,\frac{1}{S^2+19/4S+9/2}$	4	Se	
$\frac{16}{5}\,\frac{S^2+19/4S+9/2}{S+14/5}$	$\frac{72K}{5}\,\frac{1}{S+14/5}$	$\Downarrow$	$\frac{16S}{5}$	Sh	
$\frac{156}{25}\,\frac{S+30/13}{S+14/5}$	$\Downarrow$	$\frac{30K}{13}\,\frac{1}{S+30/13}$	$\frac{156}{25}$	Se	
$\frac{507}{40}\,(S+30/13)$	$\frac{117}{4}\,K$	$\Downarrow$	$\frac{507S}{40}$	Sh	
$\frac{117}{4}$	$\Downarrow$	K = 1	$\frac{117}{4}$	Se	

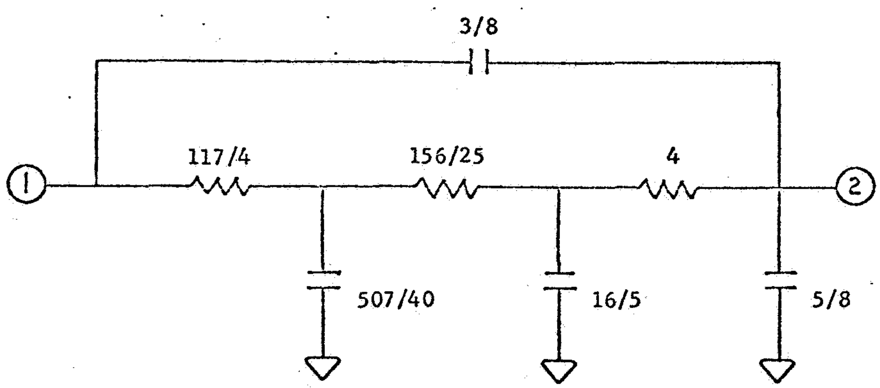


Figure 2-20. Tabulation of the development and the network schematic for Example 2-11.

CHAPTER 3

A COMPARISON WITH OTHER METHODS

This chapter presents alternate solutions to network realization problems (using the method of Chapter 2) which have been solved previously as illustrative examples of the various methods to be discussed. The author assumed during the development of the networks presented in this chapter that maximum network gain and a low number of elements were (in the order stated) the most important design criteria. Other criteria were ignored. For example, no attempt was made to obtain a particular network structure. In all of the networks developed, the maximum network gain was achieved. In addition, the resulting networks require fewer elements than what was required by the earlier solutions.

3.1 The Fialkow-Gerst Networks.

If T satisfies the conditions given in Eqn. 1-19, Fialkow and Gerst [9] have shown that T can be realized by an RC 3 T.N. Their realization procedure consists of splitting the network into two parallel networks each of which has a peculiarity that enables it to be simplified. Then each of the two remaining networks can be treated as a new problem which is less complex than the original. The Fialkow-Gerst method is completely general. The gain constant, K, is exactly specified beforehand. Unfortunately, the method often calls for an excessive number of elements.

For example, consider the network produced by Fialkow and Gerst in [9] for the transfer function:

$$T = \frac{(S^2-S+9)(S+3)}{(S+1)(S+3)(S+14)} \,. \qquad (3\text{-}1)$$

Their original solution is given in Figure 3-1. It requires an unnecessarily large number of elements.

Later in [10] Fialkow gave a different solution with fewer elements to an almost identical problem:

$$T = \frac{(S^2-S+9)(S+3)}{(S+1)(S+3)(S+12)} \quad \text{and} \quad y_{22} = \frac{(S+1)(S+3)(S+12)}{(S+2)(S+6)} \,. \qquad (3\text{-}2)$$

Fialkow's network is pictured in Figure 3-2.

The procedure of Chapter 2 was applied to the set of parameters given in Eqn. 3-2. The result is shown in Figure 3-3. The transfer function in Eqn. 3-1 could be realized in the same manner. The removal coefficients would be different, but the network structure the same. The realization in Figure 3-3 requires four fewer elements than the solution given by Fialkow.

The first two steps in the development require a special explanation. A second order high pass ladder was removed first to eliminate the poles of y_{22} and y_{21} at infinity. Regardless of what coefficient is chosen as the residue of y_{22A}/S at $S=-6$, the same gain reduction step will be necessary after the $\wp$ removal. Following

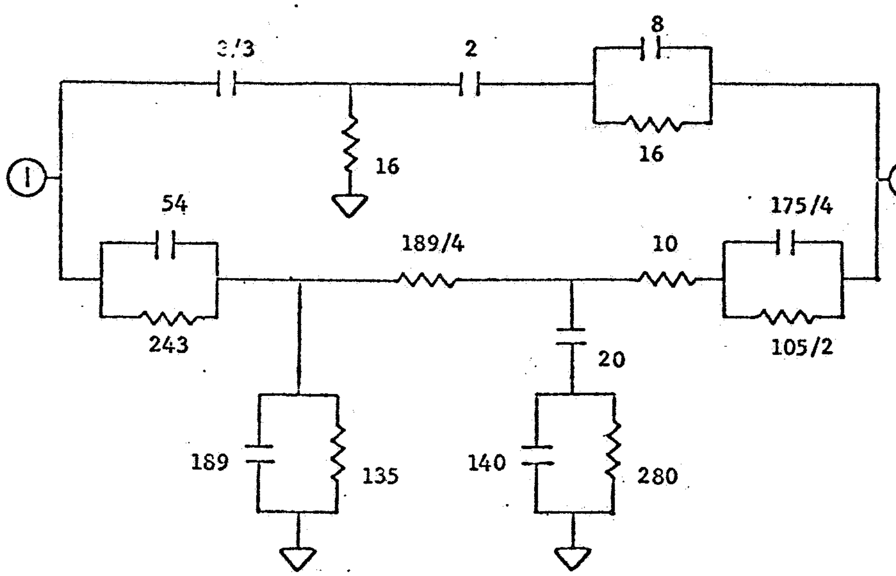


Figure 3-1. A Fialkow-Gerst network for the transfer function given in Equation 3-1.

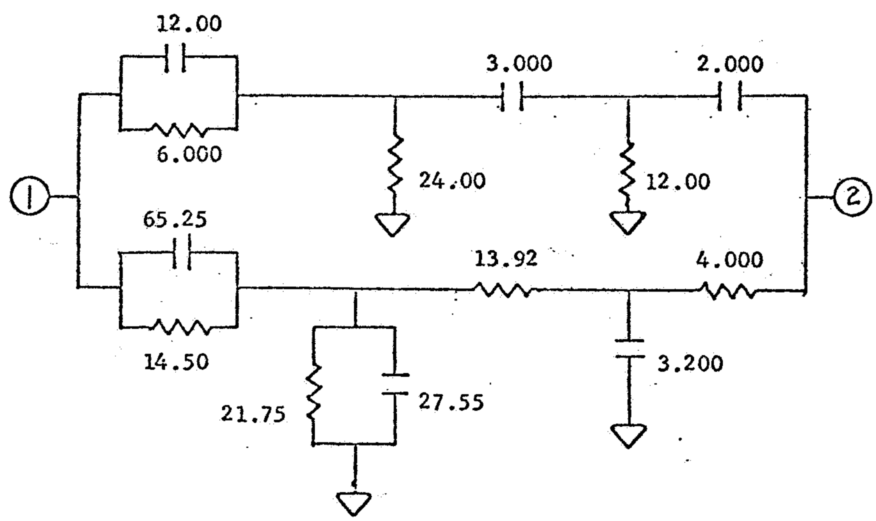


Figure 3-2. Fialkow's network for the parameters given in Equation 3-2.

y_{22}	$-y_{21}$	T	y	type	N
$\frac{(S+1)(S+3)(S+12)}{(S+2)(S+6)}$	$\frac{K(S^3+2S^2+6S+27)}{(S+2)(S+6)}$	$\frac{K(S^3+2S^2+6S+27)}{(S+1)(S+3)(S+12)}$	$S + \frac{9S/4}{S+6}_H$	P	A
$\frac{23}{4}\ \frac{S^2+6S+144/23}{(S+2)(S+6)}$	$6K\ \frac{S+9/2}{(S+2)(S+6)}$	$\frac{(24K/23)(S+9/2)}{S^2+6S+144/23}$	$\frac{3}{4}$	Sh	
$\frac{5(S+6/5)(S+9/2)}{(S+2)(S+6)}$	$\Downarrow$	$\frac{6K}{5}\ \frac{1}{S+6/5}$	$\frac{22}{5}(S+\frac{9}{2})$	Se	
$5\ \frac{S+6/5}{S+26/11}$	$6K\ \frac{1}{S+26/11}$	$\Downarrow$	5	Se	
$\frac{275}{64}(S+6/5)$	$\frac{165}{32}K$	$\Downarrow$	$\frac{275}{64}S$	Sh	
$\frac{165}{32}$	$\Downarrow$	$K = 1$	$\frac{165}{32}$	Se	
$\frac{S(S+33/4)}{S+6}$	$\frac{KS^2}{S+6}$	$\frac{KS}{S+33/4}$	$\frac{11}{8}S$	Se	A
$\frac{11}{3}(S+33/4)$	$\frac{11}{3}KS$	$\Downarrow$	$\frac{121}{4}$	Sh	A
$\frac{11}{3}S$	$\Downarrow$	$K = 1$	$\frac{11}{3}S$	Se	A

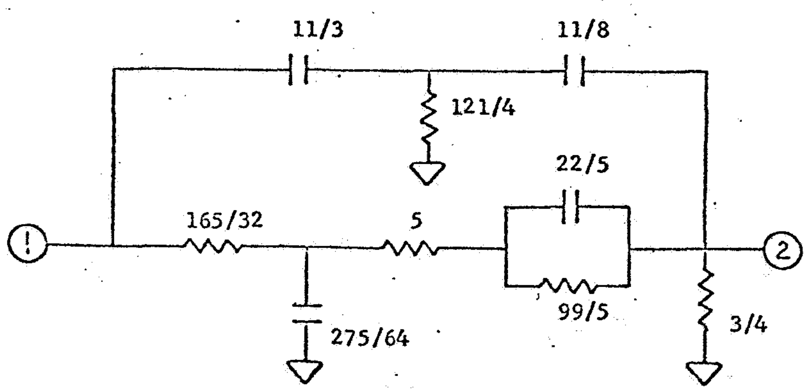


Figure 3-3. Tabulation of the development and the network schematic for the parameters of Equation 3-2.

the procedure outlined in Example 2-3 (second solution), a shunt conductance removal was chosen for the gain reduction step. The value of the conductance was calculated from: $g_0 = y_{22}(0)\left(1-T(0)\right)$, which gives $g_0 = 3/4$.

With this value in mind, and remembering that it is independent of the above mentioned residue, the residue was chosen so that after the P removal, the shunt removal would not only perform the necessary gain reduction, but would also shift a zero of y_{22} to $S = -9/2$ which produces a common zero at $S = -9/2$. The value of this residue was determined from:

$$\left.\left(\frac{(S+1)(S+3)(S+12)}{(S+2)(S+6)} - S - \frac{aS}{S+6} - \frac{3}{4}\right)\right|_{S=-9/2} = 0,$$

which gives $a = 9/4$. If this special choice of residue had not been made, an additional element would have been required.

The remainder of the development is just realization of a ladder network in the usual manner.

3.2 Cascade Realizations.

The cascade approach to the RC 3 T.N. realization problem can also be thought of as an extension of the zero-shifting procedure. If there are no complex transmission zeros, the usual result is a ladder network. To realize complex zeros, networks known to have a pair of complex zeros (e.g. the 'bridged tee' network) are placed in cascade or tandem with the remainder of the realization. Usually, only minimum phase transfer functions (i.e. transmission zeros are all in the left half plane) are considered.

The following problem was solved in [25] using a cascade technique:

$$T = \frac{(S+2)(S+3)(S^2+6S+18)}{(S+8)(S+9)(S+10)(S+11)} \quad . \tag{3-3}$$

The resulting network is given in Figure 3-4. A solution with five fewer elements, obtained by the method of Chapter 2, is given in Figure 3-5. Since the degree of the denominator of Eqn. 3-3 is four, Figure 3-5 gives a minimum capacitor solution.

The network in Figure 3-5 was developed in the same manner as the network in Example 2-8. Note that the network structures are identical even though Example 2-8 had two pairs of complex transmission zeros.

Both solutions to Eqn. 3-3 given here possess an unusual spread in the order of magnitudes of element values. This spread is attributed to the relative closeness of the four poles of T [25]. In order to obtain the order of accuracy shown in Figure 3-5, it was necessary to carry out the calculations to a higher degree of accuracy than what is indicated.

3.3 The Successive π-T Realizations.

An interesting method of realization has been given by Ozaki [28] and Lucal [23]. It consists of alternate π and T network removals. This procedure allows specification of all three y

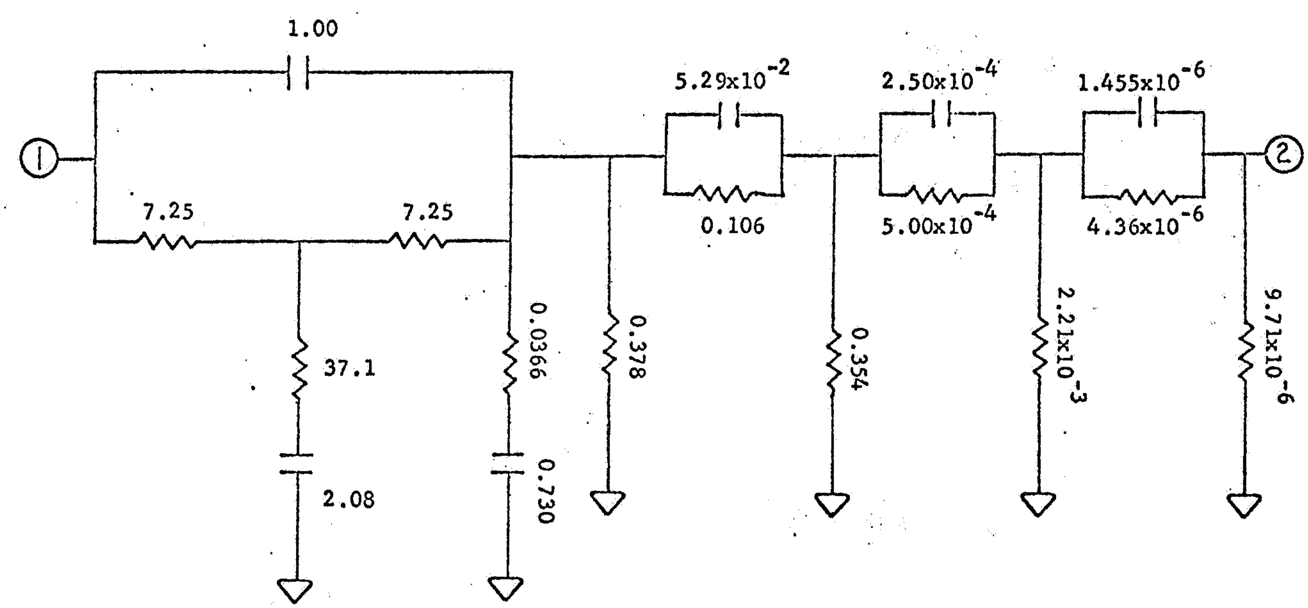


Figure 3-4. A cascade realization of the transfer function given in Equation 3-3.

y_{22}	$-y_{21}$	T
$10^{-6}\frac{(S+8.00)(S+9.00)(S+10.0)(S+11.0)}{(S+8.40)(S+9.40)(S+10.4)}$	$10^{-6}K\ \frac{S^4+11.0S^3+54.0S^2+126S+108}{(S+8.40)(S+9.40)(S+10.4)}$	$K\ \frac{(S+2.00)(S+3.00)(S^2+6.00S+}{(S+8.00)(S+9.00)(S+10.0)(S+}$
$10^{-6}\ \frac{NNC}{(S+8.40)(S+9.40)(S+10.4)}$	$10^{-6}K\ \frac{S(S^3+10.868S^2+50.291S+91.269)}{S^3+28.200S^2+264.08S+821.18}$	$K\ \frac{S(S^3+10.868S^2+50.291S+91.}{NNC}$
$10^{-6}\ \frac{S(S^3+28.355S^2+267.02S+835.05)}{(S+8.40)(S+9.40)(S+10.4)}$	↓	$K\ \frac{S^3+10.868S^2+50.291S+91.}{S^3+28.355S^2+267.02S+835}$
$60.218\times10^{-6}\ \frac{S^3+28.355S^2+267.02S+835.05}{S^2+18.998S+89.863}$	$60.218\times10^{-6}K\frac{S^3+10.868S^2+50.291S+91.269}{S^2+18.998S+89.863}$	↓
$60.218\times10^{-6}\ \frac{NNC}{S^2+18.998S+89.863}$	$60.218\times10^{-6}K\ \frac{S(S^2+9.8528S+30.996)}{S^2+18.998S+89.863}$	$K\ \frac{S(S^2+9.8528S+30.996)}{NNC}$
$60.218\times10^{-6}\ \frac{S(S^2+19.063S+90.481)}{S^2+18.998S+89.863}$	↓	$K\ \frac{S^2+9.8528S+30.996}{S^2+19.063S+90.481}$
$8.8167\times10^{-3}\ \frac{S^2+19.063S+90.481}{S+9.5962}$	$8.8167\times10^{-3}K\ \frac{S^2+9.8528S+30.996}{S+9.5962}$	↓
$8.8167\times10^{-3}\ \frac{NNC}{S+9.5962}$	$8.8167\times10^{-3}K\ \frac{S(S+6.6228)}{S+9.5962}$	$K\ \frac{S(S+6.6228)}{NNC}$
$8.8167\times10^{-3}\ \frac{S(S+9.6341)}{S+9.5962}$	↓	$K\ \frac{S+6.6228}{S+9.6341}$
$2.2434\ (S+9.6341)$	$2.2434K\ (S+6.6228)$	↓
$2.2434\ (S+6.6228)$	↓	$K = 1$

Figure 3-5(a). Tabulation of the development for the transfer function given in Equation 3-3.

$-y_{21}$	T	y	type	N
10 $\frac{S^4+11.0S^3+54.0S^2+126S+108}{(S+8.40)(S+9.40)(S+10.4)}$	$K\frac{(S+2.00)(S+3.00)(S^2+6.00S+18.0)}{(S+8.00)(S+9.00)(S+10.0)(S+11.0)}$	0.13152×10^{-6}	Br	
$)^{-6}K\frac{S(S^3+10.868S^2+50.291S+91.269)}{S^3+28.200S^2+264.08S+821.18}$	$K\frac{S(S^3+10.868S^2+50.291S+91.269)}{NNC}$	9.5131×10^{-6}	Sh	
$\downarrow$	$K\frac{S^3+10.868S^2+50.291S+91.269}{S^3+28.355S^2+267.02S+835.05}$	$1.0169\times10^{-6}S$	Se	
$.218\times10^{-6}K\frac{S^3+10.868S^2+50.291S+91.269}{S^2+18.998S+89.863}$	$\downarrow$	61.160×10^{-6}	Br	
$).218\times10^{-6}K\frac{S(S^2+9.8528S+30.996)}{S^2+18.998S+89.863}$	$K\frac{S(S^2+9.8528S+30.996)}{NNC}$	4.9841×10^{-4}	Sh	
$\downarrow$	$K\frac{S^2+9.8528S+30.996}{S^2+19.063S+90.481}$	$60.632\times10^{-6}S$	Se	
$.8167\times10^{-3}K\frac{S^2+9.8528S+30.996}{S+9.5962}$	$\downarrow$	28.478×10^{-3}	Br	
$8.8167\times10^{-3}K\frac{S(S+6.6228)}{S+9.5962}$	$K\frac{S(S+6.6228)}{NNC}$	54.653×10^{-3}	Sh	
$\downarrow$	$K\frac{S+6.6228}{S+9.6341}$	8.8515×10^{-3}	Se	
$2.2434K(S+6.6228)$	$\downarrow$	6.7555	Sh	
$\downarrow$	K = 1	2.2434S+14.858	Se	

development for the transfer function given in Equation 3-3.

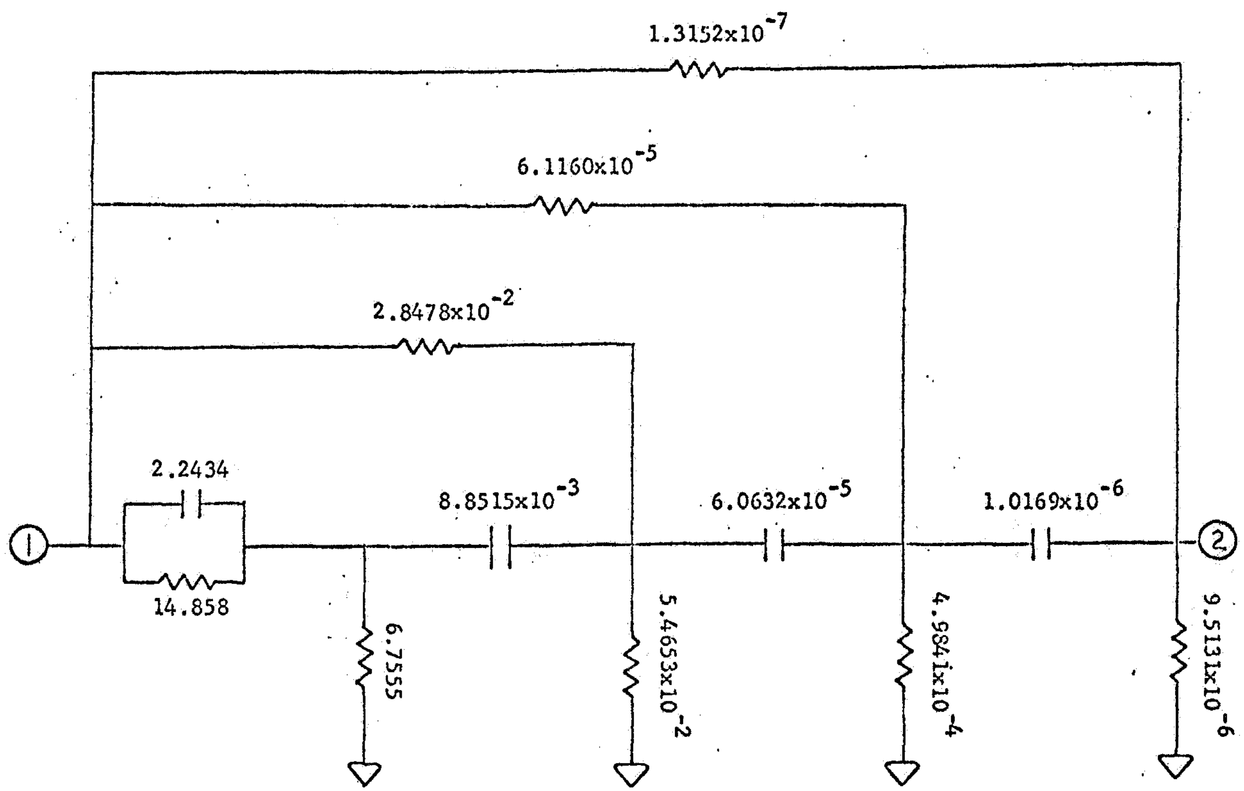


Figure 3-5(b). The network schematic for the transfer function given in Equation 3-3.

parameters: y_{22}, y_{21}, and y_{11}. The general structure that can be expected from application of the method is shown in Figure 3-6.

Since the method of Chapter 2 does not yield a prespecified y_{11}, it is somewhat unfair to compare its results to Lucal's networks. However, by neglecting y_{11} and using the y_{22} and y_{21} given by Lucal [23] in his first example, the exact network given by Lucal was arrived at using the method of Chapter 2. The details of the development and the network schematic are shown in Figure 3-7. In [23], Lucal also gives a second network for the same parameters, but it requires two additional elements.

Another example given by Lucal has the parameters:

$$y_{22} = \frac{36S^4+533S^3+1572S^2+1183S+36}{36(S+1)(S+2)(S+3)} \text{ and } -y_{21} = \frac{(S^2+1)(S^2+S+1)}{(S+1)(S+2)(S+3)}. \qquad (3\text{-}4)$$

Lucal has developed two networks for these parameters (y_{11} was also specified). The networks are shown in Figure 3-8.

The method of Chapter 2 gives a network with five fewer elements for the parameters in Eqn. 3-4; however, the resulting y_{11} is different than the y_{11} specified by Lucal. The network development and schematic are given in Figure 3-9.

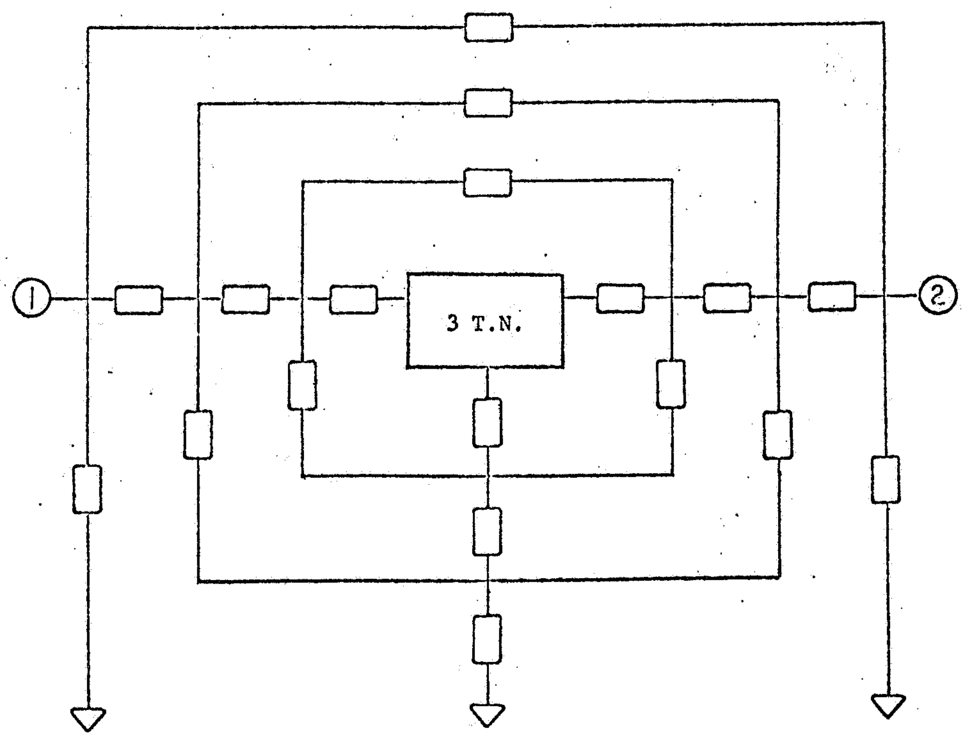


Figure 3-6. The successive Pi-Tee realization.

y_{22}	$-y_{21}$	T	y	type	N
$\frac{S^3+9S^2+14S+2}{(S+1)(S+2)}$	$K\,\frac{S^3+3S^2+6S+2}{(S+1)(S+2)}$	$K\frac{S^3+3S^2+6S+2}{S^3+9S^2+14S+2}$	S	Br	
$6\,\frac{S^2+2S+1/3}{(S+1)(S+2)}$	$4K\,\frac{S+1/2}{(S+1)(S+2)}$	$\frac{2K}{3}\,\frac{S+1/2}{S^2+2S+1/3}$	6	Se	
$6\,\frac{S^2+2S+1/3}{S+5/3}$	$4K\,\frac{S+1/2}{S+5/3}$	⇩	$\frac{30}{7}S$	Sh	
$\frac{12(S+1/2)(S+7/3)}{7(S+5/3)}$	⇩	$\frac{7K}{3}\,\frac{1}{S+7/3}$	$\frac{132}{149}(S+\frac{1}{2})$	Se	
$\frac{33}{7}(S+7/3)$	11K	⇩	$\frac{33}{7}S$	Sh	
11	⇩	K = 1	11	Se	

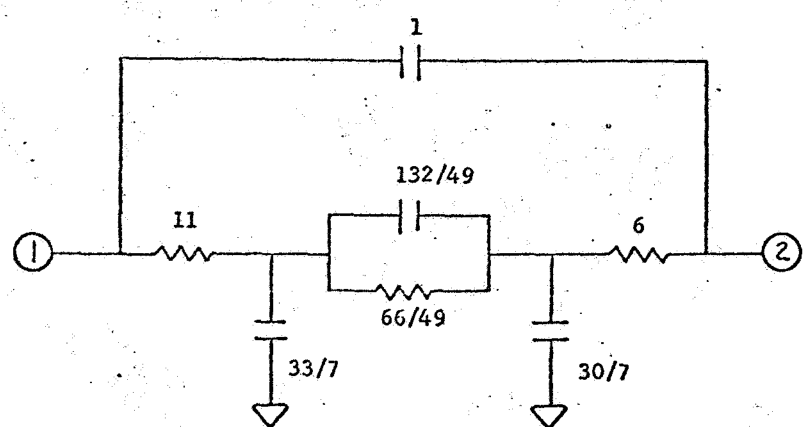


Figure 3-7. Tabulation of the development and the network schematic for Lucal's [23] first example.

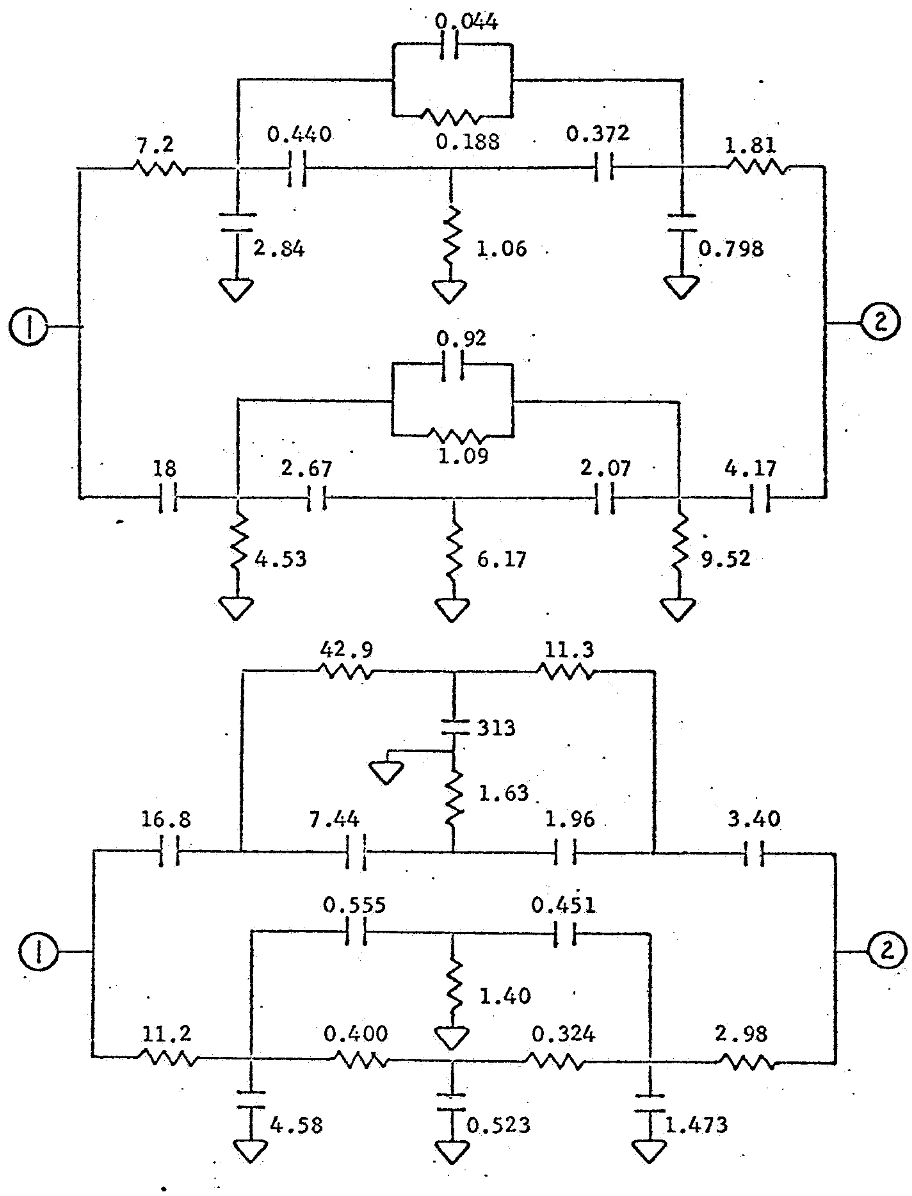


Figure 3-8. Realizations of the parameters specified in Equation 3-4 given by Lucal [23].

y_{22}	$-y_{21}$	T
$\frac{36.00S^4+533.0S^3+1572S^2+1183S+36.00}{36.00(S+1.000)(S+2.000)(S+3.000)}$	$K \frac{S^4+S^3+2S^2+S+1}{(S+1.000)(S+2.000)(S+3.000)}$	$36K \frac{(S^2+1.000)(S^2+1.000S+1}{36.00S^4+533.0S^3+1572S^2+1}$
$\frac{36.00S^4+497.0S^3+1386S^2+937.0S}{36.00(S+1.000)(S+2.000)(S+3.000)}$	$\frac{KS(S^3+1.000S^2+1.8333S+0.16667)}{(S+1.000)(S+2.000)(S+3.000)}$	$\frac{36.00K(S^3+1.000S^2+1.8333S+0}{36.00S^3+497.0S^2+1386S+937.0}$
$1.2996 \frac{S^3+13.806S^2+38.500S+26.028}{S^2+3.6616S+2.7614}$	$1.2996K \frac{S^3+1.0000S^2+1.8333S+0.16667}{S^2+3.6616S+2.7614}$	$\downarrow$
$1.2996 \frac{NNC}{S^2+3.6616S+2.7614}$	$1.2996K \frac{S(S^2+0.93965S+1.6123)}{S^2+3.6616S+2.7614}$	$\frac{KS(S^2+0.93965S+1.6123}{NNC}$
$1.2996 \frac{S(S^2+4.3801S+3.9880)}{S^2+3.6616S+2.7614}$	$\downarrow$	$K \frac{S^2+0.93965S+1.6123}{S^2+4.3801S+3.9880}$
$4.2254 \frac{S^2+4.3801S+3.9880}{S+2.0439}$	$4.2254K \frac{S^2+0.93965S+1.6123}{S+2.0439}$	$\downarrow$
$4.2254 \frac{NNC}{S+2.0439}$	$4.2254K \frac{S(S+0.15080)}{S+2.0439}$	$\frac{KS(S+0.15080)}{NNC}$
$4.2254 \frac{S(S+2.4289)}{S+2.0439}$	$\downarrow$	$K \frac{S+0.15080}{S+2.4289}$
$26.657 (S+2.4289)$	$26.657K (S+0.15080)$	$\downarrow$
$26.657 (S+0.15080)$	$\downarrow$	$K = 1$
$\frac{S+1/6}{S+1}$	$\frac{K}{6} \frac{1}{S+1}$	$\frac{K}{6} \frac{1}{S+1/6}$
$\frac{6}{5} (S+1/6)$	$\frac{K}{5}$	$\downarrow$
$\frac{1}{5}$	$\downarrow$	$K = 1$

Figure 3-9(a). Tabulation of the development for the parameters given in Equation 3-4.

$-y_{21}$	T	y	type	N
$K \dfrac{S^4+S^3+2S^2+S+1}{(S+1.000)(S+2.000)(S+3.000)}$	$36K \dfrac{(S^2+1.000)(S^2+1.000S+1.000)}{36.00S^4+533.0S^3+1572S^2+1183S+36.00}$	$\frac{1}{6}+\frac{5}{6}\frac{S}{S+1.000}$ L	P	A
$\dfrac{KS(S^3+1.000S^2+1.8333S+0.16667)}{(S+1.000)(S+2.000)(S+3.000)}$	$\dfrac{36.00K(S^3+1.000S^2+1.8333S+0.16667)}{36.00S^3+497.0S^2+1386S+937.0}$	4.3380 S	Se	
..7[illegible]K $\dfrac{S^3+1.0000S^2+1.8333S+0.16667}{S^2+3.6616S+2.7614}$	↓	0.078436	Br	
$1.2996K \dfrac{S(S^2+0.93965S+1.6123)}{S^2+3.6616S+2.7614}$	$\dfrac{KS(S^2+0.93965S+1.6123)}{NNC}$	12.171	Sh	
↓	$K \dfrac{S^2+0.93965S+1.6123}{S^2+4.3801S+3.9880}$	1.8768 S	Se	
$4.2254K \dfrac{S^2+0.93965S+1.6123}{S+2.0439}$	↓	3.3332	Br	
$4.2254K \dfrac{S(S+0.15080)}{S+2.0439}$	$\dfrac{KS(S+0.15080)}{NNC}$	4.9113	Sh	
↓	$K \dfrac{S+0.15080}{S+2.4289}$	5.0213 S	Se	
26.657K (S+0.15080)	↓	60.727	Sh	
↓	K = 1	26.657S+4.0199	Se	
$\frac{K}{6} \; \frac{1}{S+1}$	$\frac{K}{6} \; \frac{1}{S+1/6}$	1	Se	A
$\frac{K}{5}$	↓	$\frac{6}{5}$ S	Sh	A
↓	K = 1	$\frac{1}{5}$	Se	A

levelopment for the parameters given in Equation 3-4.

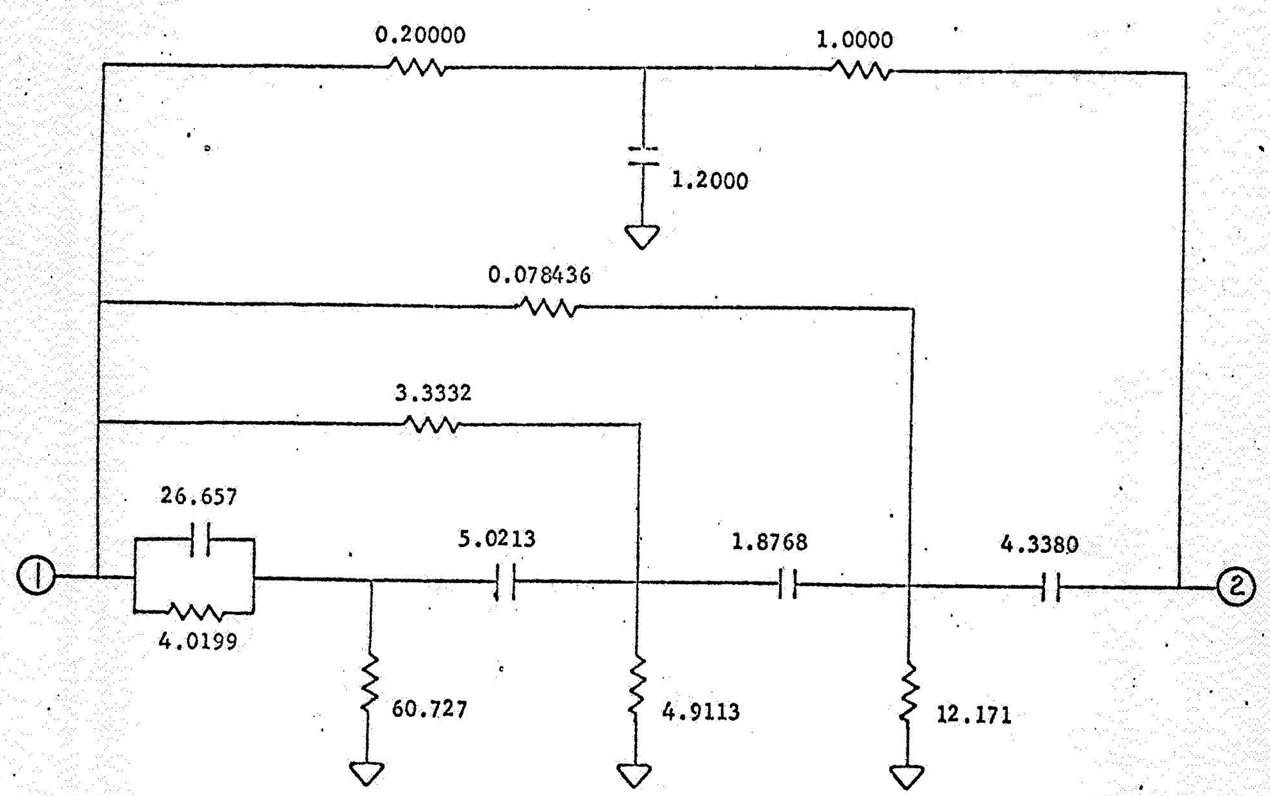


Figure 3-9(b). The network schematic for the parameters given in Equation 3-4.

CONCLUSION AND RECOMMENDATIONS FOR FURTHER STUDY

This thesis has presented an effective RC three-terminal network realization procedure. The procedure can be applied whenever one or all of the parameters y_{22}, y_{21}, and T are specified. The realization procedure is not a completely general method of RC 3 T.N. transfer function synthesis since the realization is completed only to within a constant multiplier K, a factor present in both T and y_{21}. However, control of the network K is usually not too difficult because of the systematic tabulation of the development. If too low a K is achieved, it is an easy matter to find the step or steps which caused the degeneration of the network gain. An alternate development can then be made.

The procedure gives the designer much control over certain features of the finished product. Not only can the designer regulate the network gain, but he exercises control over items such as the network structure and the number of elements. The procedure gives the designer an unusual amount of insight into how he can add elements to obtain a higher gain, or change the network structure to save elements, etc.

Chapter 3 has indicated that the method can produce solutions which are competitive with networks obtained by some other methods -

especially if a low number of elements is desired. The author believes that this justifies investigation into the following areas:

(1) Development of a computer program which would produce a variety of networks for a given set of parameters. For a high order realization problem, the circuit designer would then be given numerous alternate networks from which to choose. The program should also have the flexibility to let the designer specify certain network configurations, the result being a network or networks with such a configuration whenever possible.

(2) Extension of the procedure to permit specification of load and source admittances. Certain load admittances can easily be designed into the network. However, because of the bridge and parallel network removals, specification of a series source admittance appears to be a difficult problem.

(3) Extension of the method to provide a realization procedure for transfer functions with complex poles. That is, development of a similar RLC three-terminal realization procedure.

LIST OF SYMBOLS

symbol . . . . explanation

Br . . . . .Bridge removal.

$\mathcal{P}$ . . . . .Parallel network removal.

Se . . . . .Series removal.

Sh . . . . .Shunt removal.

K . . . . .Network gain constant.

σ . . . . .Real part of S.

S . . . . .Complex variable.

T . . . . .Forward voltage transfer function.

y_{ij} . . . . Conventional short circuit admittance parameters.

3 T.N. . . . . . .Three-terminal network.

APPENDIX

The rules for shifting zeros and poles of y_{22} using the shunt and series type of removal are easily verified by application of the well-known root locus technique. In this appendix the first rule for shunt removals will be derived. The other rules follow by using similar arguments.

The question to be answered is: "How are the zeros of y_{22}' related to the zeros of y_{22} whenever part of a pole of y_{22} is removed via a shunt type removal?" Assume that the pole is located at $S = -b$. Then y_{22}' and y_{22} are related by the following expression:

$$y_{22}' = y_{22} - \frac{aS}{S+b} ,$$

$$\text{where:} \quad a < \frac{S+b}{S} y_{22} \Big|_{S=-b} .$$

The impedance $1/y_{22}'$ is an RC driving point impedance. Consequently, its poles and zeros alternate along the negative real axis. A pole is located at or nearest to the origin. The poles and zeros of $1/y_{22}'$ can be plotted in the S plane. The result will be a plot such as the one shown in Figure A(1).

Let us now estimate where the zeros of y_{22} are located with respect to the zeros of y_{22}'. Rewriting the equation which relates the two admittances gives:

$$y_{22} = y_{22}' + \frac{aS}{S+b} .$$

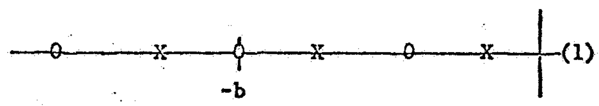
The zeros of y_{22} can be determined if the zeros of:

$$y_{22}' \left[1 + \frac{aS}{(S+b)y_{22}'} \right] \qquad \text{are known.}$$

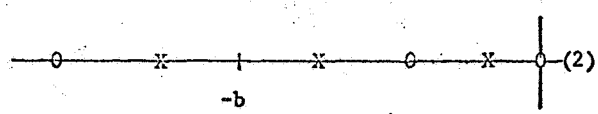
If a = 0, then the zeros of y_{22} are identical to the zeros of y_{22}'. As a increases from zero, the zeros of y_{22} can be estimated by examining the root locus of $S/(S+b)y_{22}'$. This locus can be determined by observing the pole-zero configuration in Figure A(2). Knowledge of this locus permits estimation of the zeros of y_{22} as shown in Figure A(3).

It can be concluded that each zero of y_{22} is further from the point S = -b than the corresponding zero of y_{22}'. Thus, partial removal of the pole of y_{22} at S = -b produces zeros in y_{22}' which are closer than the zeros of y_{22} to the point S = -b. This establishes rule (1) given in the shunt removal discussion of Section 2.1.

$\frac{1}{y_{22}'}$ $\begin{cases} \text{zeros} = 0 \\ \text{poles} = X \end{cases}$



$\frac{S}{(S+b)y_{22}'}$ $\begin{cases} \text{zeros} = 0 \\ \text{poles} = X \end{cases}$



zeros of y_{22} = $\square$

X's and O's are same as in (2)

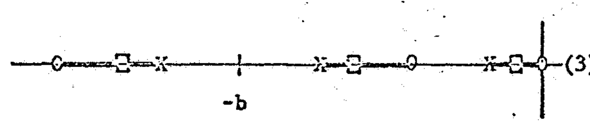


Figure A. (1) Pole-zero plot of $1/y_{22}'$. (2) Pole-zero plot of $S/(S+b)y_{22}'$. (3) Zeros of y_{22} estimated from (2).

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